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Phenomenology of Points Mathematics

Eric Fredua-Kwarteng*
University of Toronto, CANADA

Francis Ahia
University of Toronto, CANADA

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Abstract: This is a preliminary paper about a large research project on social mathematics. It proposes points mathematics, a variant of social mathematics, as a viable context for teaching mathematics to adults. Points mathematics, focuses on observing, representing and investigating patterns, regularities and quantitative relationships stemming from convertible points, that businesses offer to their customers/clients for the purpose of encouraging loyalty and for boosting up sales in competitive markets. Using ten illustrative examples, the paper asserts that points mathematics provides practical, realistic context for teaching fundamental mathematics concepts and skills to adult students. These include, but not limited to, the four operations of mathematics (addition, division, subtraction and multiplication), variable, linear equation, graph, rates, percent, ratio, patterns and proportion. The paper is grounded in the theory of realistic mathematics education (RME), that posits that the teaching and learning of mathematics should be contextually-based; entails explaining and solving contextual problems; and establishing high-level interactive relationship between learning and teaching. The paper concludes with three recommendations to guide mathematics teachers of adults who want to implement points mathematics as part of their mathematics curriculum. However, the paper is the first phase of a large research project that explores social mathematics and how it could be integrated in mathematics curricular contents for adult students.

Keywords: *Points mathematics, social mathematics, realistic mathematics education, adults learning mathematics, context.*

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Introduction

In Ontario and across Canada, adults enroll in mathematics for college and university preparation, upgrading for high school diploma, and for job/workplace training programs. In most of these mathematics classrooms, pedagogy is based on abstraction in which classroom activities and other assignments are merely regurgitation of mathematics formulas, notations and contexts that have little relevance to or meaning in either the adult students' life-world or that of their communities. As a result, most adult students regard mathematics as emotionally boring and irrelevant to living a meaningful, fulfilling life (Crustafsson & Mouwitz, 2003).

As we live in the information and technological age, we can observe many emergent quantitative phenomena in our society. In fact, mathematics teachers who are able to connect mathematics teaching, learning, and assessment to quantitative phenomena such as points schemes would make it more relevant a pursuit for their adult students. Fortunately, mathematics teachers have enormous influence in their classrooms to make that connection (Kelly, 2019).

Purpose and Rationales

This paper proposes that points mathematics as an emergent quantitative phenomenon in our society should be incorporated into basic adult mathematics education curriculum. Points mathematics focuses on observing, representing and investigating patterns, regularities and quantitative relationships stemming from convertible points. Businesses offer these convertible points to their customers/clients for the purpose of encouraging loyalty and for boosting up sales in competitive markets.

The proposal is made for four major reasons. First and foremost, incorporation of points mathematics into mathematics learning and teaching has the potential to make mathematics interesting, relevant and realistic to adult students. The

* **Corresponding author:**
Eric Fredua-Kwarteng, University of Toronto, OISE, 252 Bloor Street West, Toronto, Ontario, Canada. ✉ efredua_2000@yahoo.ca

interest that it would generate in adult students along with the sense of relevance of mathematics could motivate them to explore mathematics in-depth.

External motivators are motivation that arises outside the individual adult student such as praises from teachers, teacher support and encouragement, attaining high grades or marks on assignment, test and examination, and associated prizes for performance, work related promotion and pay increases. This is in contradistinction to internal motivators which originate inside the individual student. Point mathematics could act as a powerful external motivator for adult students who have internalized the notion that mathematics has nothing to do with real-life issues, situations and has no value outside the crucible of the school house (Herges et al., 2017; Povey, 2010).

Third, researchers have found that personalization of mathematics- connecting it to people's daily lives and activities--has positive effects in terms of students' mathematics learning growth and interest in mathematics learning (Smeets, 2018). Moreover, points mathematics would help adult students to understand the application of mathematics concepts in real-life situations. This is crucial in that adult students live in a social environment where social mathematics is deeply embedded in numerous life activities (Evans et al., 2017).

Finally, social empowerment through mathematics has been defined as having control over one's life in the worlds of work, life and social affairs (Ernest,2002). Points mathematics as a pedagogical strategy has the likelihood to lead to social empowerment of adult students in our social environment that is becoming increasingly numerate.

The paper is outlined as follows. The first part provides a brief review of the literature on approaches to teaching basic mathematics to adults. The second part highlights the theory of realistic mathematics education (RME) which undergirds the paper. The third part is a discussion of social mathematics of which points mathematics is an integral part of it. It goes on to discuss the significance of points mathematics. The fourth part, the methodology, provides ten illustrative examples of points mathematics to demonstrate its significance and the range of mathematical concepts that are applicable in that context. The fifth part focuses on a brief discussion of the illustrative examples of points mathematics.

The last part is the recommendations and conclusion. It offers three recommendations to make the teaching of points mathematics personally relevant and meaningful to adult students. The conclusion reiterates the pedagogical benefits of points mathematics. It also reveals the next phase of our research agenda on social mathematics.

Literature Review

Pedagogical approach for teaching mathematics to adults has been a subject of continuous debate in the literature. Colwell et al.(1998) present three-dimension theoretical approaches for teaching basic mathematics to adult students: personal construct of mathematics, procedural mathematics and abstract mathematics. With regard to the personal construct of mathematics approach, teachers validate personal experiences, knowledge and skills that adults bring to mathematics learning.

Pedagogically, teachers solicit from their adult students real quantitative situations they are confronted with in their lives, how they deal with them and ask them to share such strategies, methods or techniques in the classroom. That is, teachers start with adult students' fund of mathematical knowledge and build the lesson on it. Thus, adult students are regarded as mathematically knowledgeable rather than mathematically deficient who need amelioration through classroom instruction.

Regarding the personal approach, Brooks (2015) has suggested that it is only effective if the mathematics teacher is prepared to act as an impartial mediator between the informal/street methods/strategies that adult students bring to mathematics learning and the formal methods/strategies that curriculum or textbooks advocate by integrating both. That implies, from our perspective, that each strengthening the weaknesses of the other.

The procedural approach to mathematics teaching is mostly used in schools and post-secondary institutions. It focuses on mastering a step-by-step sequence of actions designed to accomplish a specific goal (Ngu & Phan, 2016). Its primary object is not to develop students' conceptual understanding of mathematics but to encourage regiments of repetition, memorization, and conformity to instructions. Students are required to mimic procedurals, rules, algorithms and to strive for the right answers or solutions. In the abstract approach, mathematics is not applied to real-life situations, though emphasis is placed on understanding numerical, geometrical and graphical relationships as well as problem-solving. Colwell et al. (1998) endorse the personal approach as the most effective way to teaching basic mathematics to adults. That approach is appealing in that it could be combined with eclectic elements from the other two approaches. For example, in problem-solving students could be encouraged to use both formal and informal methods or algorithms. As well, the focus of the personal approach on everyday real-situations could be a source of great motivation to adults who want to develop practical knowledge and tools for dealing with quantitative events in their lives and those of their families, friends and neighbours.

Another debatable approach to mathematics teaching to adult students is the socio-cultural orientation. It recognizes mathematics learning as a social practice in specific contexts with values, beliefs and relations (Gerger, 2014). This

approach to mathematics teaching challenges the long-held view of mathematics as decontextualized, abstract and value-free discipline with enormous power for transfer across different aspects of life and activities (Gerger, 2014). This approach suggests that mathematics is taught as practices, not a mechanical skill without contextualization. Mathematics teaching is grounded in everyday practices of mathematics and moves to formal practices only with explicit communication to students (Baker, 2009). Consequently, this approach shares similar characteristics with the personal approach that Colwell et al. (1998) have advocated.

Several researchers emphasize the importance of the socio-cultural approach for teaching mathematics to adult students (Baker, 2009; Baker, et al., 2008; Evans et al., 2013; Gerger, 2014). The principal reason is that adults live in a numerate environment in which they unavoidably use mathematics to solve or make sense of everyday life problems (Crustafsson & Mouwitz, 2003; FitzSimons & Godden, 2000; Hoyles et al., 2000; Kelly, 2019; Viskic & Petocz, 2006; Wedege et al., 1998; Wedege, 1999).

Adults as Wedege (2010) has alluded engage in a range of social practices such as employment, seeking employment, parenting and caring for dependants, budgeting, organizing and doing shopping, dealing with banking transactions, and preparing for voting in elections. Certainly, these are the normative contexts in which adults utilize mathematics and for that matter mathematics teaching and learning should be grounded in those contexts. For instance, using debit and credit associated with banking transactions will assist adult mathematics learners to make practical sense of negative and positive numbers (Brooks, 2015).

Other group of mathematics education researchers have suggested that adult learners of mathematics have a strong preference for concrete approach that offers them opportunities to utilize methods from their everyday life experiences (Balomenou & Totsis, 2018; Morgan, 2017; Nonesuch, 2006; Swanson & Williams, 2004). This contrasts with abstract orientation with its emphasis on formalism such as procedures, notations, formulas and equations. Nevertheless, the two approaches are not mutually exclusive as they share more in common than differences. For both approaches, real-life situations, materials and organizations are used to promote the understanding of abstract mathematical processes, concepts or ideas. Brooks (2015) supports this observation by asserting that adult learners of mathematics do not necessarily prefer their mathematics teacher to start from concrete situations and move into abstraction; teachers could also start from abstraction and move to real-life situations. It all depends on what students find comfortable in relation to their learning preference and style.

The utilization of everyday life activities to promote understanding of abstract mathematical concepts becomes increasingly relevant when we look at the nature of mathematics. For this paper, mathematics may be encapsulated as any human activity that focuses on identifying patterns, relationships and solving problems using numerical magnitude, logic, symbols, language, and technological devices.

Swanson and Williams (2014) also offer the following definition of mathematics which shows its intimate connection to real-life , "a modeling of practice which abstracts from the patterns and regularities of real practical experience in the world" (p.200). Though the modeling could be taken to any high level of abstraction, its connection with real-life or practical experience still remains intact.

As a dynamic human invention, mathematics serves as an important instrument for commerce, aesthetics, recreation, scientific and technological advancement (Budd, 2018; Denvir et al.1982). In fact, Budd (2018) has emphasized that the present civilization with its ubiquitous technological sophistication would have been impossible without mathematics. Consequently, mathematics has become an indispensable part of human civilization.

Theoretical Perspective

The theory of Realistic Mathematics Education (RME) informed the paper. It is a domain specific instructional approach to mathematics education. In RME theory, learning mathematics implies doing mathematics of which solving contextual problems is a critical part of it. Other characteristics of RME include giving students opportunities to invent mathematics concepts and methods of problem-solving; and making the learning-teaching relationship interactive (Fauzan et al. 2002).

The development of RME can be traced to the work of mathematician and philosopher Hans Freudenthal of the Netherlands. Freudenthal (1971) asserts that mathematics should be conceptualised as a human activity similar to the way mathematicians conceive it, involving problem-solving, looking for problems and organizing problems (see also Gravemeijer, 1994). Accordingly, Freudenthal (1973) dismisses the notion of mathematics as a mere body of knowledge, rules and algorithms for students to consume and reproduce when needed in classroom assignments, tests and examination.

Freudenthal emphasized the importance of real-world context which according to him provides the source for identification and formulation of mathematical problems and further mathematization. Without contexts, he argues that mathematics learning is a meaningless intellectual exercise. It should be added that in translating RME into practice, real-world context does not exclusively imply existing issues, situations and matters of tangential interest to

the students. It also includes mathematizing relationships and processes of significant matter to the students (Gravemeijer & Doorman, 1999).

Freudenthal (1991) distinguished between two key concepts: Horizontal mathematization and vertical mathematization in the following way:

"Horizontal mathematization leads from the world of life to the world of symbols. In the world of life one lives, acts (and suffers); in the other one symbols are shaped, reshaped and manipulated, mechanically, comprehendingly and reflectingly. This is vertical mathematization. The world of life is what is experienced as reality (in the sense I used the word before) as is a symbol world with regard to abstraction. To be sure the frontiers of these worlds are vaguely marked. The world can expand and shrink also at one another's expense" (p.42).

Though there is a body of generally accepted principles, concepts, notations, and theorems that guide mathematics learning, mathematics is basically a human activity that entails mathematics modeling- mathematizing of activities, patterns and events occurring at the physical as well as in the social environment. Such activities may involve applying mathematics to solve problems or to gain valuable insights into social, economic and political phenomena.

Over the decades, researchers have attributed many benefits to RME such as its effectiveness for improving students' problem-solving ability (Yuanita et al. 2018); for improving the mathematics participation rates and achievement of low-attaining students (Barnes, 2004; 2005); teaching algebra (Theodora & Hidayat, 2018); and for improving general mathematics teaching and learning effectiveness (Gravemeijer & Terwel, 2000). Recently, it has been argued that RME is an effective theoretical framework for developing learning materials for mathematical problem-solving (Ulandari et al. 2019).

To conclude this section, it should be noted that the use of the words "context" and "realistic" in discussing the theory of RME is likely to cause some confusion. The meaning of these words can be encapsulated in a single sentence: any mathematical activities within the physical and mental imagination of students qualify as contextual and realistic (Dickinson & Hough, 2012). Accordingly, some puzzles, stories, and factitious situations could be viewed as contextual and realistic in applying RME to mathematics learning.

Social Mathematics

Social Mathematics is any activity involving numerical calculations, ordering by numerical magnitude or other forms of quantitative reasoning performed on a daily basis by masses of people in society. It also includes interpretation of numerical data about the structure and functioning of economic, political and social institutions of a nation-state (Thompson, 2006). The former definition is adopted for the purpose of this paper. Examples of that variant of social mathematics are shopping (which may involve adding numerical totals; subtracting to cross-check change received; multiplying prices with quantities where repeat purchases occur; comparing prices of similar items with regard to style, size, colour, texture; calculating discounts, accumulating or redeeming points, etc.); buying fast-fast foods or eating at restaurants; choosing a cell phone plan, reading sports league tables; buying gas or bus transit tickets; and payment of taxes(including sale tax).The following are the salient features of social Mathematics:

1) It is not restricted to a specific profession or vocation. Masses of people from all walks of life participate in it, mostly as consumers, clients or citizens;

2) It requires the application of the four fundamental operations of mathematics: addition, subtraction, multiplication, and division. Skills in ratios, rates, proportion, fractions, percentage, reading of graphs and tables, and simple algebra and patterning are also important in doing social mathematics; and

3) It is done on a daily basis by masses of people. No one could live his or her life in our society without having to do one or other forms of social mathematics.

Points mathematics is an integral part of social mathematics as it involves businesses offering convertible points to their customers for the purpose of encouraging loyalty and boosting up sales in competitive markets. This genre of social mathematics is distinguished from others such as making numerical data or statistics relevant to society; applying mathematics to social issues; using mathematics to empower disadvantaged segments of society.

As well, points are different from discounts and bonuses that businesses offer to their customers or clients. Discounts are one-time, direct reduction from the list prices of products or services. An example is 40% off the listed price of a product in a clothing store. Bonus is also a one-time incentive that involves offering customers/clients additional units of products at reduced price or free of charge after specified purchases are made. Buy one item and gets one more free or buy two items and get the third one free of charge or 50% off the price; buy a service and get free-of-charge installation are examples of bonus offerings.

In Canada and other Western countries, for the past two decades or so points have joined bonus and discount as customer loyalty tools that businesses apply to sustain or boost-up sales revenue or volume.

Businesses that offer points specify how customers/clients can earn and redeem their earned points for cash, services or products. Other businesses offer direct cash points based on the amount of money customers spend at the specific time of transactions. For this reason, customers/clients have to read carefully the terms and conditions of any points offering schemes and understand how the rules operate; namely how to earn and redeem points.

Significance of Points Mathematics

Historically, organized sports such as soccer, football, basketball and volleyball have used points to rank teams in league tables according to total points earned. For instance, in a soccer league the winner (the one that scored more goals) of a game gets 3 points and the loser gets 0. Goal differential is the total goals scored minus goals conceded. In the case of a draw each team gets 1 point. So in terms of symbols W = Win, L= Loss, and D or T= Draw or Tie, and GD = goal differential which may be either positive or negative.

These days some businesses offer their customers/clients points either for a short-term or long-term duration. They issue points cards to their customers/clients that are swiped electronically each time the customers make purchases. This allows points earned as well as points redeemed to be recorded in an electronic format. In this case the following formula is important:

Opening balance plus current points earned minus points redeemed equal current points balance. Mathematically, this is the same expressed in symbols: OB+ CPE - PR = CPB. This relation holds true for calculating any missing parts of points records.

Businesses that offer points include restaurants, grocery stores, gas stations, companies, electronic communication providers (e.g. cell phone, land phone, tv, internet), financial Institutions (e.g., banks, credit unions), hotels, amusement centres and theaters, airlines and gas stations. It is important that adult students should learn how to compute and redeem points as consumers/clients; to compare points offerings of different businesses and choose the one that fits their special social or financial needs and circumstances.

Further, points have become an important motivation tool for attracting and retaining customers/clients in the business world. As some adult students are likely to become future entrepreneurs, they have to learn how to design better points programs relative to those of other competing businesses or in terms of their own notion of what constitutes fair business practice.

Furthermore, pedagogically, social mathematics of points is one of the most effective ways to help adult students to understand and apply the fundamental operations of mathematics and other essential mathematical concepts. It provides a way for illustrating practical applications of mathematical concepts such as ratio, percent, fraction, proportion and linear algebra.

Moreover, the big ideas of mathematics themes are the fulcrum of mathematics. They consist of proportion, number sense, measurement, variable, relations, representation, induction/deduction and balance (Glasser, 2011). Points mathematics offers numerous practical opportunities for the big ideas of mathematics themes to be incorporated in mathematics teaching and learning.

Lastly, almost every adult is familiar with points schemes offered by retailers, banks, fast-food outlets, and gas stations. Learners' familiar context is considered an effective strategy to facilitate mathematics learning and the possibility of making it fun to learn (Morgan, 2017). This is consistent with the personal and socio-cultural approaches to mathematics teaching and learning.

Methodology- Ten Illustrative Examples

In the examples below, points in mathematics activities are illustrated to show their social and personal connectivity. These real-life examples were selected from various business organizations and we designed activities for each in accordance with our professional experience with adult students.

The questions we ask in the illustrated examples or the activities we have designed are flexible enough to allow students to experience the practical nature of mathematics. However, the names of the organizations connected with the illustrated examples and other details have been omitted to protect the identities of those organizations.

Example 1 A grocery store offers points to its customers based on certain products they buy. The store allows customers to redeem their points only for groceries. Here are the Store's rules for redeeming points:

- Customers can not redeem points for cash;
- Customers get points for selected items per transaction;
- Points can only be redeemed for an equivalent value of groceries.
- 20000 points are equivalent to $20 worth of groceries and could be redeemed;

● Points earned cannot be transferred to another person except spouses.

a) This week Mr. Smith has a total of 68 000 points on his card. What is the maximum amount of money can he redeem for groceries?

b) Mr. Smith also earned 17000 points this week. What is his net balance on his points card after he redeemed the maximum amount?

c) In one weekly fryer the store offers its customers 40000 points for every $20 spent on produce. How much are the points in terms of cash equivalent? If a customer spends $60 on produce, how much would he/she save on the transaction?

d) According to the Store's rules, 200000 points are equivalent to $20 worth of groceries. Express this relation in reduced ratio form and explain what it means.

Example 2 Erica owns a courier business. From January to March 2016, her points statement from a gas station company is shown below:

Balance brought forward?

Jan 1. 590 points

3 redeemed 1200points

7. 540

8. 450

15. 380

20. 400

25. 290

31. 580

Feb 4. 450

8. 600

15. 240

18. 310

24. 300

29. 400

March 2. 250

10. 140

15. 200

20. 210

26. 340

29. 280

31. 360

Total Points=

a) Calculate the points brought forward for Erica.

b) Erica can only redeem her points for car wash. If one car wash requires 500 points, how many times can Erica wash her car using her current total points?

Example 3. A bank pays 1% cash back to its customers for any amount spent using their credit cards. For the table below complete the cash back amount for Melissa:

Date. Amount Spent. Cash Back

March 3. $299.3 4

Feb. 14. 312.63

May 25. 69.47

Jan. 10 9. 99

June 10. 307.92

Dec. 9 17.00

a) Based on the above table, how much total cash-back did Melissa receive from the bank?

b) What is the difference, if any, between calculating 1% of the total amount spent for the six months and calculating 1% of each amount spent and adding them up?

Example 4. A bank provides its credit card holders the following points conversion table:

Points	Cash
4300	$25
8600	50
17200	100
43000	250
86000	500

a) If Ms. Havana has earned 100500 points, how much cash will she receive by redeeming her points?

b) Mr. Jamison has 4200 points. How much can he get in cash?

c) 4300 points equal $25. Express this in reduced rate form.

d) Using a calculator divide the points on each line by their corresponding cash amount. What did you notice?

e) Ms Taylor redeemed her points and received $750. How many points did she redeem?

f) Mr. McKnight has accumulated 4290 points. How much maximum cash can he get?

Example 5. A drug store provides the following reward table:

Points Required.	Cash Value
8,000.	$10
22,000.	30
38,000.	60
50 000.	85
95,000.	170

a) Hector has 17800 points on his points card. He redeemed some points and received $10. How many points are left on his points card!

b) Jennifer has been able to accumulate 118 000 points. How much cash value is her points?

c) What patterns do you notice in the table? Explain the pattern using numbers, words or symbols of your choice.

Example 6. In a soccer league of 15 teams each team is required to play a total of 30 matches (15 home matches and 15 away matches) during the season. A win = 3 points; a tie or draw =1 point and a loss =0. As you read the league table, answer the following questions:

a) Warrior United lost 5 matches and tied 7 matches. How many points did it receive at the end of the playing season?

b) Shining Stars won 20 of its matches and tied 3 matches. How many matches did it lose? How many total points did it get at the end of the playing season?

c) If a team must earn at least 35 points to avoid relegation, would Brilliant City Club face relegation if it ties 20% of its matches?

Example 7. Ms. Jonson has a health food store that sells all kinds of health products including vitamins and minerals. There are other health food stores located in the shopping mall as Ms. Jonson's. Ms. Johnson is thinking about boosting up sales so she designed points cards for her customers. Any time a customer buys products worth $50 or more in one transaction, the

customer gets 5% off the total price before taxes as cash points. Customers have the option to accumulate their cash points and redeem them at a later date. Ms Jonson plans to give the points cards to 400 of her customers.

a) Assuming each of the 400 customers bought products worth $50 or more in the month of Sept, how much minimum would Ms Jonson spend on sales promotion?

b) Kelly, Ms. Johnson's daughter, thought her Mom spends too much money on sales promotion and wants to redesign the points scheme to make it more effective and cost efficient. Help Ms. Johnson's daughter to redesign the sales promotion scheme.

Example 8. Bompato money coupon is given out to customers for purchases paid by cash or credit card.It is based on the pre-tax total purchases at the rate of 4% or 0.04cents for every dollar spent. In the month of January, Janetta bought the following items from Bompato Store:

Juice extractor….. $149.99

Windscreen wipers.. 61.99

Firestone tires…….520.00

Car washer liquid… ...49.00

Janetta gave the cashier Bompato money coupon worth $15 toward the purchases.

(a) How much Bompato money coupon will she receive?

(b) How much will she pay for her purchases in total after taking into consideration the money coupon she gave to the cashier?

Example 9. Airplane Miles is a popular loyalty program in Canada with over 5million subscribers across the country. Points collected can be redeemed for travel, merchandise and discount at stores. Airplane Miles can be collected through:

√ Spending at participating stores

√ Airplane Miles credit cards

√ Buying directly from Airplane Miles.ca

Normally, 1 reward mile is equivalent to every $20 spent. There are two Airplane Miles credit cards:

AM Elite Golden card Eligibility criteria:

● Good-Excellent credit score

● Minimum annual personal income of $60,00

● Annual fee: $130

Benefits of AM Elite credit card:

● 1 reward mile for every$10 anywhere

● 20% discount on Airplane Miles flight within North America

● Welcome bonus of 4000 miles

● Quarterly airport lounge passes

● Comprehensive insurance coverage

Canadian Supper Credit card Eligibility criteria

✓ Good-Excellent credit score

✓ Minimum personal annual income: $0

✓ Annual fee: $110 (waived in the first year)

Benefits of CSCredit card

Benefits

✓Welcome bonus of 2000 miles (points)

✓No annual fee

✓ 1 reward mile for every $20 spent at

participating stores

✓ 25% off at National and Alamo car rental

locations world wide

Airplane Miles Points Required for a flight

Low Season. High S

Toronto-New York City. 1500. 1800

Vancouver- San Diego. 2600. 3100

Calgary- San Francisco 3000. 4000

Winnipeg-Cancum. ……5000. 7500

Montreal- Paris ……..7000. 10000

(a).Which of the two Airplane Miles credit cards do you prefer? Give reasons for your preference.

(b). Tina has AM Elite Card with 10000 miles. She intends to travel from Montreal to Paris. What is the cash equivalent of 10000 miles?

©. Smith has Canada Super credit card and in three months he made the following purchases from participating stores:

January……. $490

February…….843

March…….229

Calculate the total Airplane Miles he earned on the three transactions.

Example 10. Temco is a retail giant in Canada and offers its customers a points scheme by which they can accumulate and redeem their points for products exclusively. Temco provides the table below to help its customers to convert their points into cash equivalents.

Temco Reward points. Redemption

Balance. Value ($)

2000…… $10

4000……20

5000……25

6000……..30

8000……40

10000……..?

20000……..?

?……250

?……300

(a) Study the table carefully. First, divide each numerical value in the first column on each line by its corresponding cash value in the second column. What did you notice?

(b) Using the idea of unit ratio or rate, we divided 2000 by 10 to get 200. This means for every dollar spent you get 200 points. Use this rate or proportional technique to derive the numbers required to fill in the missing numbers in both columns.

© Since 200 is constant and multiplying it by the dollar amount gives the required points we have the following relationship:

200 x dollar amount = Required points.

In symbolic terms we have the following equation:

200D = R, where D represents the dollar amount and R represents the required points.

Use the equation to calculate the cash equivalent of 85000 points.

(d) A customer has suggested that Temco should give 2000 bonus points to any customer whose points balance at the end of the year is at least 50000. She argued that the bonus points would be a great motivation for customers to achieve that target. She also stated that accumulated points are deferred expense for Temco, for which customers should be compensated. State your reasons for agreeing or disagreeing with the customer.

A Short Discussion of Examples

The ten illustrative examples indicate that points mathematics provides opportunities for both horizontal and vertical mathematization. Horizontal mathematization as it has been stated is concerned with transforming problems found in real-life into symbols, while vertical mathematization relates to the process of abstraction. Though all the questions involve horizontal mathematization, other questions could be posed that have implications for abstraction or vertical mathematization.

As a matter of fact, the illustrative examples demonstrate that, points mathematics provides a rich content for adult students to learn practical applications of rudiments of the four operations of mathematics, order of operations, variable, formula, fractions, rate, ratio, percent, proportion and equation (or balance).For instance, example 2 is not only about addition and division operations but little algebra is also involved.

As well, points mathematics as demonstrated in the examples will allow adult students to understand that mathematics is inseparable from everyday life, contrary to popular perception (Mendick, 2015). In addition, example 7 provides a space for self-expression through designing points schemes that are deemed fair from a consumer/client or business perspective. Example 10 (d) also offers a space for expression of personal point of view. This is crucial in the sense that communication is an integral part of mathematics learning (Leonelli, 1999).

Besides, points mathematics provides opportunities for adult students to discover mathematical ideas and concepts through exploration of different situations and real-world problems relating to points schemes. Both examples 4, 5, 9 and 10 provide opportunities for exploration of several mathematics concepts such as graphing versus tables (or charts); the concept of x and y-intercepts; using graphs for estimating or interpolation, extrapolation; calculating the slope of a graph or rate of change; and deriving equation of a straight line. Of significance is that the tables in examples 4, 5, 9 and 10 could be graphed using either a graph sheet or graphing calculator. And any values, points or money could be conveniently read from the graph.

The tables in the three examples also provide excellent practical opportunities to teach the concepts of sets, relations and functions that are considered fundamental building blocks for advanced mathematics such as calculus. Therefore, the power of points schemes is not only limited to teaching and learning basic mathematics but also advanced mathematics.

Another important issue that arises from the illustrative examples is that businesses invariably provide a table rather graph to show points and their equivalent cash values. The fact is that tables make it easy to compare pairs of related values; they are easily understood and almost everybody can read them. By contrast, despite their visual effects graphs tend to display the overall trend of a phenomenon. However, this observable issue could be assigned to adults as a mathematics project.

Recommendations and Conclusion

Based on our literature review and the analyses, we offer the three recommendations below to guide mathematics teachers of adult students:

(1) Mathematics teachers have a professional obligation to make mathematics personally relevant and meaningful in order to motivate adult learners to invest the time and effort to learn it. Accordingly, the mathematics teacher should be knowledgeable and understand the social background of adult learners and why they want to learn mathematics;

(2) Retailers offer various point schemes; thus, mathematics teachers have a range of choices to select point schemes that are personally and socially meaningful to the adult students and design appropriate activities for them; and

(3) Personalization of mathematics learning and teaching requires that the mathematics teacher recognizes and validates any informal computational methods and conceptualization of mathematics concepts that adult students bring to the classroom. The recognization and validation involves the teacher combining the "formal" and the informal methods and allowing one to complement the other in ensuring efficiency, meaningfulness and proficiency computation and coordination of thoughts.

Our in-depth knowledge of the literature of adult mathematics learning, along with the examples of points schemes we have illustrated in this paper, we are strongly convinced that points mathematics has great potential to promote adult understanding of mathematics concepts. The illustrative examples are ample evidence that point schemes offer multiple opportunities for adult mathematics students to learn practical applications of mathematics concepts. Finally, as part of our research agenda on social mathematics, we have planned to experiment points mathematics with two

groups of adult students enrolled in a university preparatory program in Canada. One group called the experimental group would be introduced to points mathematics, after that their level of mathematics interest, confidence, attitudes and their perception of its relevance to life would be measured. These measurements would be compared to the control group.

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