Research Article https://doi.org/10.12973/ejmse.1.2.91

European Journal of Mathematics and Science Education

Volume 1, Issue 2, 91 - 106.

ISSN: 2694-2003

https://www.ejmse.com/

Preservice Secondary Mathematics Teachers' Actional Beliefs about Teaching Geometric Transformations with Geometer's Sketchpad*

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Received: July 5, 2020 ▪ Revised: September 23, 2020 ▪ Accepted: December 12, 2020

Abstract: Preservice mathematics teachers' beliefs about actions related to the use of the technological tools in teaching mathematics may affect how they are going to use them in their classroom activities. However, there is a limited evidence of what beliefs they hold on their intended actions of using technological tools in teaching mathematics. This study presents two preservice high school mathematics teachers' actional beliefs related to their intended actions in teaching geometric transformations (GTs) using Geometer's Sketchpad (GSP). The study comprised of a series of five task-based qualitative interviews with each of two senior undergraduate preservice teachers at a medium-sized public university in the Rocky Mountain Region of the United States. This study used a radical constructivist grounded theory (RCGT) with five assumptions—symbiosis, voice, cognition, adaptation, and praxis as a theoretical framework to guide the study process. The thematic findings of the study included four in vivo categories of their beliefs associated with actions of teaching GTs with GSP – assessment of student learning, engaging students in a group activity in exploring GTs with GSP, engaging students in individual activity in exploring GTs with GSP, and exploring GTs with GSP as 'suck it up and do it.' Pedagogical implications of these categories have been discussed.

Keywords: *Preservice mathematics teachers' beliefs, technology integration, radical constructivist grounded theory (RCGT), actional beliefs, Geometer's Sketchpad (GSP), geometric transformations.*

To cite this article: Belbase, S., Panthi, R. K., Khanal, B., Kshetree, M. P., & Acharya, B. R. (2020). Preservice secondary mathematics teachers' actional beliefs about teaching geometric transformations with Geometer's Sketchpad. *European Journal of Mathematics and Science Education*, *1*(2), 91-106. https://doi.org/10.12973/ejmse.1.2.91

Introduction

The phrase 'Actional Belief' seems to be a philosophical domain that implies having intentional beliefs to act on something or act in a certain way or use something for an intentional outcome. The kind of beliefs preservice mathematics teachers hold depends on the target or object of the belief. These beliefs can be "concrete, abstract, metacognitive, and subjective" (Lao & Young, 2020, p.8). The actional belief is subjective because it is very personal and private. However, the consequence of most of the subjective beliefs is translated into actions. "In this sense, our beliefs constitute the seeds of our actions, and different beliefs yield different actions" (Lao & Young, 2020, p. 13).

Falk (2004) states, "If one has a complex of actional beliefs and desires, one's actions are such as to maximize expected value in terms of them." Although the context of this statement is different in Falk (2013) than in this study, however, the proposition can be linked with preservice mathematics teachers' actional beliefs about using GSP in teaching GTs in terms of their intentionality and desire to apply the tool in a certain way or do not apply the tool based on their expected values or potential outcomes. Hence, the actional beliefs associated to the regiment of the linguistic intentionality can also be interpreted in the context of mathematics teacher education to observe preservice teachers'

* This study was a part of the doctoral dissertation of the first author.

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cognitive, affective, and mathematical dispositions. Therefore, actional beliefs are an individual's disposition to act in a certain way in a particular situation. Other rare citations of actional beliefs are mentioned in a few pieces of literature; for example, Ihde (2000) and Buckareff (2006). This paper explores preservice mathematics teachers' actional beliefs associated with using the GSP as a tool for teaching GTs with a variety of intended classroom actions or activities. However, actional beliefs can be the product of cognitive, affective, and behavioral components of preservice teachers' attitude toward the tool (e.g., GSP) and its usability in teaching mathematics contents (e.g., GTs) (Peker & Ulu, 2020).

Past research findings indicated that teachers' beliefs affect the use of technological tools (Ertmer, 2006). This view is also supported by O'Neal et al. (2017), stating that the teacher's beliefs influence the use of technology in teaching and learning. Technological tools not only help in the demonstration of mathematical constructions and proofs, but also, they aid in making it enjoyable to the learners with a more profound sense of problem-solving, creative thinking, and reasoning (Ertmer, 2006; Ertmer et al., 2012). Technological tools can help teachers and students make abstract mathematical concepts visual and dynamic otherwise impossible to see with formal equations or formulas (Belbase, 2016; Foley & Ojeda, 2007). There are various uses of technological tools in mathematics classrooms – for example, teachers construct interactive tasks in computers to engage students in problem-solving, they can create online quizzes, they can create video lessons, and even use different applications available to teach mathematics in a meaningful way (Garry, 1997). For this to happen effectively, at first, the teachers need to change their mindset about the use of various technological tools in teaching and learning mathematics, without which it may not be possible to bring changes in mathematics education (Chai et al., 2011; Leatham, 2002; Wachira et al., 2008).

There are several studies on teacher beliefs; however, the results are not consistent (Ertmer et al., 2012; Leatham, 2007; Lin, 2008). Many researchers constructed categories of teacher beliefs about technology integration in mathematics education. Some named them as 'no technology beliefs,' 'pre-mastery belief's', 'post-mastery beliefs,' and exploratory beliefs' (Misfeldt et al., 2016); and Erens and Eichler (2015) named teachers based on their beliefs as initiators, explorers, reinforcer, and symbiotic collaborators. Even others categorized teacher beliefs about technology integration as instrumental and empowerment (Chen, 2011).

Mishra and Koehler (2006) integrated pedagogical content knowledge with technology and introduced the new paradigm as technological pedagogical and content knowledge (TPACK). This construct has a significant impact on training teachers to enhance the effective use of technological tools in mathematics classrooms (Belbase, 2016; Hunter, 2015). The introduction of technology in mathematics education helps students to solve 'nonroutine tasks' using the tools for exploring, connecting, and thinking of mathematical problems and path to solutions (Cuevas, 2010). Hence, technological tools' effective use may help develop positive beliefs about mathematics, teaching mathematics, and learning mathematics (Misfeldt et al., 2016).

Most of the past studies on teacher beliefs about technology integration focused on the functional beliefs based on technological tools such as technology for algebraic thinking, technology for critical thinking, technology for developing mathematical relations and structures, and technology for contextualizing mathematics (Polly, 2015). However, there is a scarcity of literature on preservice mathematics teachers' actional beliefs in general and actional beliefs about using technological tools in teaching mathematics in particular. In this context, this study aimed to explore preservice secondary mathematics teachers' actional beliefs about teaching geometric transformations (GTs) with Geometer's Sketchpad (GSP). The research question for the study was - What action related beliefs do preservice secondary mathematics teachers hold about teaching geometric transformations using Geometer's Sketchpad? The research question can be justified with the four principal functions of technology— "sense-making, reasoning, problem-solving, and communication" (National Council of Teachers of Mathematics [NCTM], 2011, p. 1). In general, it is expected that the use of technology enables students to "visualize the results of varying assumptions" (Common Core State Standards Initiative, 2010, p. 3). The research question explored the preservice mathematics teachers' beliefs associated with the students' actions/functions with GSP while learning GTs.

Teacher Beliefs about Technology Integration

Teachers' beliefs impact instructional practices in the classroom, and as a consequence, it affects students' conceptual learning and academic achievement (Pajares, 1992; Philipp, 2007; Thompson, 1992; Yurekli et al., 2020). Yurekli et al. (2020) conducted a study on teaching mathematics for conceptual understanding to explore the relationship between teachers' beliefs and practices disclosed that teachers had a positive belief about making a strong bond to encourage conceptual understanding in their classrooms. Teachers believing in the technology were using it more frequently for the benefit of the students (Blackwell et al., 2013; Inan & Lowther, 2010).

An analysis of mathematics teachers' beliefs about technology integration from the published research reports indicated three fundamental belief paradigms – constructivist beliefs, instructivist beliefs, and integral beliefs (Table 1). These belief paradigms have been dominant characters of mathematics teachers' beliefs, and subsequently, we can see their applications and consequences. Some mathematics teachers believe in teacher-led instruction who prefer drill-and-practice less likely to use technology in mathematics teaching and learning (Teo et al., 2008). That means these teachers prefer direct teaching with the lecture method with repeated practice. They may value using technology less, except for using it under challenging computations (Ertmer et al., 2012). For them, technological tools in mathematics may be a good thing only for fast computations. They seem to give less emphasis on meaningful teaching. These teachers may undermine the importance of using technology in mathematics education (Erens & Eichler, 2015). Whereas the teachers who believe the constructivist approach seem to use technology for student-centered learning through student collaboration, inquiry, and teamwork in problem-solving (Ertmer et al., 2012). Therefore, the use of technology by mathematics teachers is significantly affected by their beliefs and attitudes towards the role of technological tools in teaching-learning (O'Neal et al., 2017).

Table 1. Beliefs about Technology Integration in Mathematics Education

Beliefs	Characteristics of Beliefs
Instructivist	The use of technological tools in mathematics teaching and learning can be very formal. The use of technology in mathematics class is rule-bound with the formalization of mathematical procedures to find correct answers. Use of technology in mathematics class follow step-by-step mechanical computations. Technology serves as a tool to solve routine mathematical problems.
Constructivist	The constructivist teachers may focus on the application of technology in mathematics education to help students construct the meaning of mathematical ideas. Students use technological tools to construct their conjectures, proofs, and explanations. Technology serves as a tool for generating new mathematical meanings and interpretations.
Integralist	The teachers who favor integralism consider that technology integration should help in teaching and learning mathematics in interactive, dynamic, creative, explorative, and active ways. Technology supports in integrated teaching of mathematics across the domains of mathematics, for example, algebra, data, and geometry together. Technological tools serve as an integral part of the new culture embedded in mathematics.

Various studies on technology integration for teaching and learning have focused on teachers' beliefs about the role of technology in teaching and learning (Deng et al., 2014; Güneş & Bahçivan, 2018; Jääskelä et al., 2017; Liu, 2011; Tondeur et al., 2008). Some of these studies have demonstrated the different uses of technology in teaching-learning mathematics. For example, according to Hopper and Rieber (1995), technology integration in education may support five stages-- utilization, formalization, integration, reorientation, and evolution. In this context, Hopper and Rieber (1995) claimed that traditional teachers, who mostly used technology for formalization and utilization, did not even reach the above integration level. Most of them seemed to use technology in teaching and learning mathematics for accuracy and speed. However, a few teachers who considered technology a means of reorientation and evolution could use these tools to transfer mathematics to other areas of applications through the use of such tools.

Technology integration in mathematics education can provide a new insight for teachers and students to think, reason, solve problems, and extend their vision beyond the classroom contexts (Cennamo et al., 2010). The creative use of technological tools may help students and teachers envision the infinite possibilities of problem-solving and reasoning in mathematics (Wilson & Cooney, 2002). Some teachers may consider their role in technology integration as a change agent to introduce new possibilities in mathematics education (Teo et al., 2008). Notably, the teachers who believe the constructivist approach to teaching-learning of mathematics may also hold a constructivist view about technology integration in mathematics education.

Some researchers discussed teacher beliefs regarding positive or negative beliefs other than traditional or constructivist (Karen-Kolb & Fishman, 2006). The teachers with positive beliefs about technology integration may use technological tools in their teaching of mathematics. They consider that technology can help them teach mathematics better compared to teaching without it. Nevertheless, there are some teachers with negative beliefs about technology use in mathematics education. They seem to consider that technology does not help them with the effective teaching of mathematics. The use of technology in a classroom has disordered the tasks with the poor organization of activities and lack of motivation due to extra effort to be needed to use the tools (Karagiorgi, 2005).

Some categories of teacher beliefs about technology integration from the past studies are- it replaces tidy paper-pencil works, the use of technology makes mathematics learning joyful, it can help students in career preparation with technological skills, and it can prepare students to look for opportunities to learn in varieties of ways (Forgasz et al., 2010). In this context, it has been agreed that technology integration provides better access to mathematics to students with equity and inclusiveness (Cennamo et al., 2010). Hence, teacher beliefs about technology integration have brought three belief profiles – traditional or instrumental, constructivist or relational, and integral or volitional.

Preservice or in-service mathematics teacher beliefs have been discussed in the forms of unproductive and productive beliefs by the NCTM (2014). The unproductive beliefs about learning and teaching mathematics focus on procedures, memorizations, standard computations, mastery of basic skills, formal structures of mathematics with rules, formulas, and definitions. The productive beliefs focus on understanding, conceptual learning, reasoning, problem-solving in context, inquiry-based mathematical tasks, active engagement in discourse and negotiation of mathematical meanings, connecting to prior concepts and experiences, and productive struggle of the learners (NCTM, 2014). These beliefs can be related to teachers' actional beliefs about technology integration in teaching-learning mathematics in general and teaching and learning of GTs with GSP, in particular.

Theoretical Assumptions of the Study

This study was guided by the ontology of nominalism and the epistemology of radical constructivism. The research methodology was guided by constructivist grounded theory. Five theoretical assumptions were conceptualized from the principles of radical constructivism (e.g., von Glasersfeld, 1978, 1995) and the methodological approach of grounded theory (e.g., Charmaz, 2006; Strauss & Corbin, 1998) to strengthen the research methodology (Belbase, 2020 & 2015). Each assumption has been discussed separately under the following sub-sections.

Symbiotic Relation

The first assumption is that the research participants and researchers have a mutualistic symbiotic relationship by integrating radical constructivism and the method of grounded theory (Charmaz, 2006; Corbin & Strauss, 2008; Steffe & Thompson, 2000; von Glasersfeld, 1995). During the construction of data through task-based participant interviews (Goldin, 2000; Maher, 1998), the research participants are not mere the source of information (data) but a collaborator with the investigator who contribute to the study through active participation in the task and think aloud in the process by the construction of participant narratives based on their experiences while going through the tasks (Steffe, 2002; Steffe & Thompson, 2000).

Researcher and the Participants' Voice

The second assumption is that research with task-based interactions carries both participants and researcher voices in the forms of vignettes, interview excerpts, personal narratives, and life stories, to name a few (Belbase, 2016). The research participants and the researcher's voices are the experiential expressions through narrative excerpts and the researcher's voice in the form of interpretive and reflexive accounts of the context, questions, and categories (Bergkamp, 2010; Hertz, 1997; Warfield, 2013). The researcher decenters his position by keeping the participants' voices upfront in the direct words of their own (Pierre, 2009), being aware of the fact that their voice may not "speak on their own" (Mauthner & Doucet, 2003, p. 418) and needed researcher's further interpretation.

Research as a Cognitive Function

The third assumption is that the research process is an active cognitive process with reorganization and re-interpretation of emergent concepts, categories, and meanings of codes and categories from data by making sense of experiences through tasks, interactions, and reflections as the cognizing subjects (Belbase, 2020, 2016 & 2015). The researcher creates a task-based situation with questions, activities, open interactions, and records everything while performing an in situ construction (Friedhoff et al., 2013). The researcher maintains a theoretical memo to keep track of the entire process with personal thinking in data analysis before administering a subsequent interview (Charmaz, 2006). All of these analytical functions are active cognitive processes (Bailyn, 1977). The series of interactions help the researcher and participants in the "organization of their experiential world" (von Glasersfeld, 1990, p. 19).

Research as an Adaptive Function

The fourth assumption is constructing new knowledge from task-based research interviews as an adaptive function throughout the research process (Lichtenstein, 2000). That means the construction of new categories undergoes through the reorganization of the codes and categories with the additional data, new context or question, and continuous analysis and interpretation during the task-based interviews (Charmaz, 2006). In this sense, the study is associated with constructing adaptive grounded categories (Layder, 1998; Welsh, 2009). The research participants and the researcher go through new experiences with additional task-based interactions with new meanings and contexts adapting to more saturated categories (Belbase, 2020 & 2015).

Fit and Viability (Praxis)

The final assumption is that the major categories or themes constructed from the task-based interview data can be examined with two praxis criteria – criteria of fit and viability by examining each emergent category from the data to see whether it resonates with the participant's expressed views (Charmaz, 2006; Glaser & Strauss, 1967; Strauss & Corbin, 1998). The researcher observes each category to examine if it constructs a viable explanation of the participant beliefs from their experiential anecdotes with "transformative possibilities of the research process and product" (Rodwell, 1998, p. 79). The researcher may seek help from others for peer debriefing and reviewing the interviews, and auditing the codes, categories, and concepts from the data (Rodwell, 1998).

This study employed these five assumptions of RCGT by outlining the participants and the researcher's role and identifying their position in the research process (Belbase, 2020 & 2015). These assumptions guided the research process that has been discussed in the next section.

Methodology

Recruitment of the Participants

An undergraduate class of twenty students in methods of teaching mathematics for preservice secondary mathematics teachers was consulted in the fall of 2013 at a medium-sized public university in the Rocky Mountain Region of the United States to inform them about the research. They were invited to participate in a series of five task-based interviews in the context of teaching GTs with GSP. Two participants were recruited for the interviews depending on their time for interviews and their interest in volunteering in the study. One of them was a male participant who had re-joined the college to get a teaching degree after many years of his private job. He did not have prior experience of using GSP besides in two sessions of methods of teaching mathematics. Another participant was a female undergraduate preservice secondary mathematics teacher aiming to be a mathematics teacher after graduation. She had prior experience of using GSP in a geometry class she took in the earlier semester.

Administration of Interviews

The first two interviews were conducted at the end of the fall of 2013 term when the research participants in the methods of teaching mathematics courses, and the rest of the three interviews were administered in the spring of 2014 when they were in their student teaching internship. Each interview episode was designed for teaching reflection, translation, rotation, and composite transformations with paper-pencil, geoboard, and GSP in a computer with task situations followed by discussions and participant reflections (Figure 1). Each interview episode ranged from 37 to 86 minutes. All the interviews were video recorded using a digital video recorder. Each interview was transcribed, analyzed, and interpreted before another interview to facilitate the construction of codes, categories, and theoretical sampling in an iterative way.

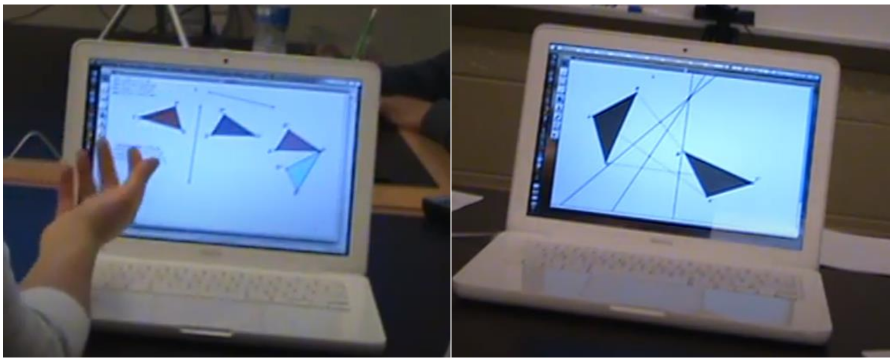

Figure 1. Cathy (a participant in the research) elaborating her beliefs about teaching geometric transformations with Geometer's Sketchpad during a task-based interview session

Writing Theoretical Memos

The researcher maintained a reflective memo after each interview to support the interview data's subsequent analysis and interpretation. These notes included significant points of discussion and questions that were discussed during the interview sessions. It also served as the observational note during the interview process with the research participants' verbal and nonverbal expressions. These theoretical memos served as a bridge between the researcher's data and reflective accounts to construct themes and categories. This process helped the researcher to reflect on the critical ideas, events, and procedures. The researcher integrated initial codes from the data to a broader concept or a category based on their similarity in meaning or concept that led to the final categories of participant beliefs.

Analysis and Interpretation

The analyses and interpretations of the qualitative task-based interview data were carried out by undertaking a classificatory analysis and interpretation of the data using the grounded theory approach (Charmaz, 2005; Strauss & Corbin, 1998). This analysis involved the construction of emergent concepts and categories from the interview data by breaking it into meaningful codes at three levels – open codes, axial codes, and selective or focused coding.

The researcher read the first two interview transcripts (first interviews with each research participant) and identified primary initial codes from the words, phrases, and sentences with significant meanings associated with the participants' beliefs. These codes served as a basis to organize the data based on a coding paradigm by characterizing and grouping these codes into primary categories known as axial codes. The axial codes supported in refining the questions for the third and fourth interviews with each participant. The third and fourth interviews with each research participant were transcribed and coded with more attention to constructing meaningful categories of the participants' beliefs about their intended actions of teaching geometric transformations with GSP. The coding paradigms from the earlier analyses were considered to align the codes and categories in a coherent way to generate meanings out of the paradigmatic structure of the secondary codes known as axial codes. The fifth interview with each participant focused on the key points and themes already generated from the first four interviews. Therefore, the coding of the transcribed data was focused on the final grouping of the codes and categories by merging similar categories together into a broader overarching category as the participants' actional beliefs.

All the layers of codes and categories were organized together into a group based on their meanings and potential connections with linkages among and across each other. These codes with similar meanings and connections to the common theme were organized under a common category to represent a thematic unit or a spectrum of the qualitative data to construct a holistic meaning (Hall, 2008). This analysis was based on a radical constructivist approach to finding deep meanings of data as a whole. The five assumptions of RCGT guided this holistic analysis and interpretive approach in the study.

Results

The preservice secondary mathematics teachers' beliefs associated with intended actions of teaching GTs with GSP had categories – assessment of student learning GTs with GSP, engaging students in a group activity in exploring GTs with GSP, engaging students in individual activity in exploring GTs with GSP, and exploring GTs with GSP as *'such it up and do it'* (Belbase, 2015). Each of these sub-categories has been discussed under a different sub-section.

Assessment of Student Learning GTs with GSP

The participants' beliefs about assessing student's learning while teaching GTs with GSP focused on both formative and summative assessment. However, their beliefs emphasized formative assessments more than the summative assessment. Their voice in Excerpt-1 (Table 2) is an example of how they expressed their actional beliefs assessing students' learning of GTs with GSP.

Table 2. Interview Excerpt-1

***Researcher**: Do you think that you can assess students' learning of GTs by using GSP?*

***Cathy**: If I get the document, it depends on what it would be. If I want the procedural method, then no, but if I want conceptual understanding, then yes, if I get the document (students' constructions), not just the printed thing, in manipulating their construction.*

***Researcher**: How would you decide that students have learned something when you teach rotation by using GSP? How will you know you are convinced of their learning?*

***Cathy**: Um, great question. I haven't really thought about the assessment of this sort of. (Pause) We can have them get a new activity completely. They will need to recreate a picture or something (else). You have a picture here (shows on a computer screen), but we want them to draw a mountain (as a model). I want them to rotate it. Everybody finds it how to do it. Send them to rotate about the lake or something else. Um, that way, I can see how they can rotate it and that how they know what rotation means, and they are not just following the procedure.*

***Jack**: You can do a formal assessment like written, or you can do if they are familiar with GSP, you can have them create it. That's gonna take a lot more time (to do it). You have to get them familiar with it and might even take it longer (time). Like, if you actually familiarize them with some of the programs (in GSP), maybe the kids are super smart, and now maybe, they catch on with that.*

***Researcher**: What will convince you that they have learned?*

***Cathy**: Um, you can print out the thing (their construction), but I mean to know that it was truly rotated (around a point). In the beautiful world, you know, we have drop boxes to put in something (to share them). Or, I can just walk up to them and ask, 'Show me what you have done?' We can do that (to see what they have done). I would ask them, like, 'where did you rotate it?', tell me, what point they rotate about. And then, say I want to rotate the top of the mountain to the left.*

***Jack**: A formal assessment like, even if you like, oral assessment they are gonna explain it to me. (Pause) Some kids can explain to me what the rotation is doing. You can do it if it is just a rotation. And if it is on paper, you can have two shapes; then, you can talk about angles of rotation. If you have GSP, they can measure it. If you have paper and a pencil, they have to use a protractor to measure it. But, if you use GSP, they can quickly measure the angles and talk about how each one goes, which is really cool. That's why I like it.*

[This interview excerpt has been taken from Belbase (2015)]

Cathy expressed that the student works on GTs with GSP could be documented and used for the assessment of their learning. Here, the documentation means written records, not only just explanation of their learning, but it may also be the real tasks they worked on and saved on their computers after they completed constructions of GTs with GSP. She appears to consider that students' activities' to be used as part of an assessment of their learning of GTs with GSP. At the same time, students' work on the construction of an object and its image under any GT may provide some clues on their learning and development. She considers that asking students to show them what they have done may also help the teacher in assessing their learning of GTs with GSP. In the same vein, Jack also seems to believe that students' tasks on GTs with GSP may be demonstrated if they are familiar with the tools in it to learn the contents and make the connection between them. He further suggests the teacher to use prompts to know students' understanding of a GT and the use of GSP. A formal assessment, including oral assessment, can be used in which the students may explain what they have done or learned to the teacher. Hence, Jack seems to view that students' explanations and their ability to manipulate the GSP to demonstrate different GTs can be used as a means to assess their learning.

Engaging Students in Group Activity in Exploring GTs with GSP

The research participants seem to believe in using group activities as a viable approach to engaging students while teaching GTs with GSP. Their views have been portrayed in the following Excerpt 2 (Table 3) as an example within this sub-category.

Table 3. Interview Excerpt-2

***Researcher**: How would you organize a group activity (while teaching GTs with GSP)?*

***Cathy**: Well, it's hard. (Pause) I think it's hard because you want both, like students to have hands-on. So, maybe, if you are gonna have a partner out of a computer, and you need to draw this, and you (another one) need to describe. Like, you are gonna describe, and you are gonna draw this one. Halfway through, you need to switch (the role). I think it's hard. Because you need to involve with it. To have a conversation, so, it would be a lot better if they all were able to do it, and have the conversation (with each other). So, with GSP, partner work is not ideal cause one person is not gonna do anything.*

***Jack**: You can have them get together and design something that involves rotation (or another GT). Like, if you have a whole block, you can really come up with something cool. Creative. (Pause) Like, have them come up with something they could realize. They can even talk about video games and how things are moving. But, having them talking about, just design something or like that, make sure that it's something that they can physically draw in GSP.*

***Researcher**: Do you think a jigsaw method works with GTs with GSP?*

***Jack**: It could. I don't know how much you can use GSP with it. But, you can totally do, like grouping them for every different transformation and having them go back and teaching in their original group. It should be fun. I mean, if they learn about all these different ones from each other, and they have combined them, that would be fun. It's time-consuming. But fun. If I have the technology, like the access, then yes. If you have something like a laptop cart and so they can move around. Jigsaw requires a lot of moving around. Also, if they are doing different things on computers, and switching computers, that's not gonna be useful. If you have access, then yes.*

***Cathy**: Yea. I think so. Like if these guys do reflections, those guys do whatever they decide and how they are gonna teach each other. You can use GSP that way.*

[This interview excerpt has been taken from Belbase (2015)]

Cathy expresses that group activity is challenging to apply in teaching-learning of GT processes with the application of GSP. She thinks that it can be challenging to engage all the students in the group-work. Such group work may not help them learn effectively. Some students can do it very nicely by learning from their peers, but it may not be a productive approach for other students because they may not participate equally in constructions and discussions with peers in a meaningful way. She is also skeptical of group activity to help all students in effective learning. She laments out that one student in a small group or a pair may do the construction while others can describe what has been constructed. She thinks that peer work can be a lot better, given that all students are able to do the construction and have a meaningful conversation. However, she doubts that only one student may be doing both (construction and explanation). She suggests switching roles in groups or with peers can be helpful in their learning. On the other hand, Jack thinks that such group-work is important for teaching-learning of GTs with GSP. He views that the teacher can engage students in the groups in designing something by using concepts of GTs with GSP. He iterates that students can create something cool that they enjoy and learn from it. He emphasizes the group members to do some constructions to study the properties of different GTs with GSP. He seems to believe that the jigsaw approach can be a helpful method to engage students in group activities. He thinks that grouping students for different transformations and letting them go back and teach other students in their original group may help each student understand different GTs.

The key points from the participants' voice are related to working with a partner in Jigsaw, having a meaningful conversation, creating different designs by using concepts of GTs with GSP in a group, forming separate groups of students for different transformations, and engaging students in self-teaching and learning from each other in the groups or peers. This result is consistent with the findings of (Gillies, 2016) which pointed out that small group performed significantly better than those working alone and students acquire more knowledge when working as a group rather than working as an individual with computer technology by making presentations, self-gauzing knowledge, peer-reviewing, and stratifying students' works in the classroom (Hodges & Conner, 2011; Santucci, 2011).

Engaging Students in Individual Activity in Exploring GTs with GSP

The participants expressed beliefs about using GSP for engaging students in individual activity in the classroom while teaching GTs. In this context, individual student activity has been considered as what each student can do in the class himself or herself without getting involved with groups. The activity can be constructions, proofs, and explanations. Excerpt 3 (Table 4) demonstrates an example of how the participants expressed beliefs about individual activities in the classroom in teaching GTs with GSP.

Table 4. Interview Excerpt-3

***Researcher**: What kind of individual activity could students in class work with?*

***Cathy**: Um, so, I think, the individual activity would be the discussion. I think that in geometry, it is incredibly important to discuss their ideas. And, because that is informal proof. And, so, saying those things out loud solidifies. In geometry, we do it a lot. I think that discussion is incredibly helpful for them. I can go up to them. And, they may ask me if they are doing this right. Then, being able to say words with their actions is really helpful.*

***Jack**: You could have them design something and spin it; if they are able to do that, some sort of real-world thing. I really like the Ferris wheel. That's really cool. I would also do a tilted wheel, which is about eight different rotations; that's really fun. It just depends on how much time you have and how you answer this. If they understand and really do stuff, that's really cool.*

[This interview excerpt has been taken from Belbase (2015)]

Cathy appears to believe student's individual discussion on ideas of GTs as a part of an activity in the class while using the GSP. In geometry, she thinks that it is incredibly important for students to discuss their ideas at the individual level. This individual discussion, for her, is a part of the informal proof of student's learning in which he or she can say things out loud. This 'think aloud' process may solidify individual thinking learning about GTs with GSP. Another point she outlines is reaching up to each student and asking him or her if he/she is doing right. Each student individually discusses their works with the teacher as a part of individual activity in learning GTs with GSP. She comments that when a student verbalizes his or her work, this can help his or her thinking and learn at a personal level through such action. On the other hand, Jack thinks that students' individual designs or constructions can be part of individual activity. He states that a teacher can have students design something with GSP and spin (rotate) it. For him, the design of an object under rotation can be related to an object in the real-world. For him, one such thing related to rotation can be a 'Ferris wheel or a tilted wheel.' He further expresses that designing a 'tilted wheel' having eight different ways of rotations can be a very interesting task for students. He agrees that the method of individual engagement of students in activities depends on different contextual factors, such as time, students' level of understanding, and their motivation in doing different kinds of stuffs with GSP.

Some of the critical points derived from the above analysis are – informal proof of learning through individual explanation, elaborating one's work as a sign of learning, and design a model of something with the GT concepts by using GSP as a part of individual activity. While doing individual activities, students can use the tutorials to engage them individually in learning the steps to use GSP to construct models of different GT processes (Wyatt et al., 1997).

Exploring GTs with GSP as 'Suck It up and Do It'

The participants shared with the investigator their beliefs about their experiences of learning GTs and using GSP in the context of teaching and learning GTs. It is considered that beliefs and affect are different psychological or mental states, and they might influence each other and inform each other and may influence actions (Philipp, 2007). The following Excerpt 4 shows their action related beliefs about teaching GTs with GSP.

Table 5. Interview Excerpt-4

***Researcher**: Tell us something about your experience of learning GSP.*

***Cathy**: Wow! Kind of let me guess. Here is your homework. Here are the proofs. Everything needs to be constructed in GSP. Get on. I think it was like ...it was so hard. I guess we didn't explore tutorial because I learned better without it. That had been helpful except those tutorials that said like 'you can go here and like do a construction of a square and like create a tool'. While you have to create, but then you don't have to go back to create a square. And so, like I had to struggle through all and kinda do all my own. People would have helped me obviously if I had asked. I remember pretty much everything I have done and I am not gonna forget it because I did it. I was able to like 'suck it up into it'. But, I don't think that a lot of my students would be up to 'suck it up and do it'. But, if they are doing this, and I do not have to teach them every single step the way they had already struggled through it, in a word if they went through it then I would be more comfortable. I guess I would not be able to do this every single time.*

***Jack**: It consists of about two class periods in the college.*

***Researcher**: How do you describe them as unfolding?*

***Jack**: We had technology presentations in methods class where each person, maybe, talk something with GSP, and that was about it. We were given step-by-step instructions. Build this and here is how you do it. You can add a little bit to it. That was very procedural and huge instructions to go to it. Very little actual hands-on.*

***Researcher**: Do you think that any conceptual experience unfolded through that?*

***Jack**: Not really. I mean it was like 'here is what you are doing. This is what it should look like, if it doesn't then you messed up'.*

[This interview excerpt has been taken from Belbase (2015)]

Cathy had an initial experience of using GSP in one of her classes. This geometry class covered different topics in geometry by using GSP. The students did not use the tutorials to learn GSP in the class, and there were no separate sessions to learn the essential tools in GSP. Therefore, they learned to use it on their own. She liked deeply engaged activity and she felt like 'sucking it up' in GSP while doing her geometry problem-solving. It seems that Cathy believes on the challenges and struggles as part of learning and teaching geometric transformations or other geometry topics with GSP.

On the other hand, Jack did not have prior experience of using GSP in geometry courses. He had a very limited experience of GSP in his course works in the preservice teacher education program. He believed that he could not use GSP because of the limited experience of it in the methods course. Although the methods class had a technology presentation activity in which each student was required to prepare and talk about GSP and other technological tools in the presentations, most of these presentations were limited on a step-by-step approach to use GSP. They were not linked with the topic of GTs. The participants' experience of using the tool in other courses might influence their anxiety about the use of the tool in teaching GTS depending on whether they possessed traditional or constructivist or even integral beliefs and conceptions about the technology and the associated contents (Peker & Ulu, 2020).

Discussion

The participants' views related to assessment are not explicit, although they stress on 'walk-around' in the classroom can be one approach to see how students are performing their tasks. However, such formative assessment may not serve the purpose of the standardized tests in mathematics, although it may help conceptual understanding (Yurekli et al., 2020). Cathy and Jack view this approach as a viable way to see if students are on task and what they are doing. According to Cole (1999), 'walking around' can be a form of informal assessment of students' learning by indirect observation. Cole (1999) states that walking around is one of the most efficient methods in a classroom that teachers can use. The teacher can use the 'on-the-spot observation' during the walk around in the class to record what the students are doing with GSP while learning GTs. Another assessment tool can be face-to-face meetings to discuss students' knowledge, skills, and understanding of GTs using GSP. The teacher can start from the student's work as a step to begin the observation of his or her confidence in using the tool in doing and thinking about GTs (Cole, 1999). Cathy and Jack's view about using students' work of construction has some viability to use the observation and conversation as an assessment tool which Cole (1999) has demonstrated its relevancy. This view is supported by Suurtamm et al. (2016) emphasized listening and responding to student thinking. The teachers should be motivated to apply different assessment techniques such as conferencing, observation, or performance tasks. In this context, Bloom's (1964) taxonomy may provide a basis to connect learning with students' tasks and their levels of understanding of GTs with GSP.

According to Siegel et al. (2008), grouping of the students' investigations of concepts and tasks allows teachers to choose the most relevant parts of their curriculum to develop a conceptual and procedural understanding of what students do and learn in a lesson. On the other hand, Muller (2010) argues that group-work facilitates a shy student to work with other students in a group and present his or her work in the group instead of the whole class. One of the major benefits of group-work, according to Bossé and Adu-Gyamfi (2011), is that students take ownership of their tasks and activities at both individual and group levels. The group-works in the classroom may help in social cohesion among the students with cooperation and collaboration in small team works (Andrini, 1991; Johnson & Johnson, 1990; Obara, 2010). In this context, four theoretical perspectives related to motivation, social cohesion, cognitive development, and cognitive-elaboration can be linked with a group activity of students while learning GTs with GSP with the theory of cooperative learning that incorporates interdependence, development, cognition, and behavior (Baloche & Brody, 2017). The group work helps in the development of group dynamics, collective identity, and group process, a sense of mutual respect, care and trust, and achievement of both group and individual goals (Johnson & Johnson 1990). Then each member of the groups may teach each other various transformations with greater and better access to GTs with GSP (Cleaves, 2008).

According to Dickinson (1994) and Manouchehri et al. (1998), the use of the technological tool (e.g., GSP) is very important for individual learning of mathematics content such as GTs to develop students' aptitudes related to content in the technological environment (such as dynamic geometry environment in GSP) (Dickinson, 1994). Students can work on the constructions, conjectures related to some proofs, and verifications of properties, and then they can go further to independent explorations of GTs beyond what is taught in the class or the course (Manouchehri et al., 1998). Both Cathy and Jack express these views in an implicit way because their expressions do not touch upon the different layers of individual activities. This seems obvious because they do not have a long experience of using GSP and have not tried it to teach GTs (until the time of the interviews) (Wyatt et al., 1997). These issues are also associated with their optimism and innovativeness and negative aspects with feeling insecurity and discomfort in using the tool (Badri et al., 2013).

It seems that the participants had limited experiences with GSP in mathematics methods or geometry classes in the preservice teacher education programs, which did not enhance their confidence in applying the tool in the teaching of GTS. Their views on limited experience and its impact on their use of the tool reflect how the intended actional beliefs about using the tool for teaching GTs are interrelated to their prior experiences. Their realization about and the level of confidence shaped their beliefs (Kuntze & Dreher, 2015). Cathy expressed that the self-learning process seemed challenging to her but rewarding for her learning experience. She was an active user and learner of the GSP and its applications in teaching geometry by self-discovering the important features and applications which enabled her to know more about the tool (Sarracco, 2005). In this sense, she developed herself as a self-directed learner in discovering the features of GSP in the foundation of geometry (Merriam, 2002), by being a responsible learner (Knowles, 1975; Tough, 1971). On the other hand, Jack did not have prior experience of using GSP in geometry or other courses. Therefore, his limited experience with GSP by step-by-step instruction was not helpful in developing a positive actional belief about the tool in teaching GTs (Cantürk-Günhan & Özen, 2010).

We can see a difference between beliefs held by Cathy and Jack teaching and learning GTs by using GSP. However, these differences were mainly due to their experiences of using GSP in other courses and how these experiences impacted their intentional action related beliefs. This also shows that one's prior experience has a significant impact on his or her beliefs about teaching mathematics in general and GTs in particular. Hence, the participants' prior experiences were critical on their beliefs and potential applications of GSP (Ng et al., 2010). Additionally, the quality of educational experiences in the teacher education programs, such as methods courses, may influence their interest, beliefs, and values toward using GSP in teaching (Martin et al., 2000). The preservice teachers' experiences of working with GSP may help them form positive actional beliefs about the usefulness and efficiency of GSP in teaching GTs (Cowen, 2009; Davis, 1989). Research has shown that several GSP learners and users learned about the tool by themselves while preparing to use it in their classroom teaching (Boehm 1997; Brumbaugh 1997; Cuoco & Goldenberg, 1997; Morrow, 1997).

Conclusion

The interview Excerpts 1- 4 presented the participants' (Cathy and Jack's) beliefs related to the assessment of student learning of GTs with GSP, engaging students in a group activity in exploring GTs with GSP, engaging students in individual activity in exploring GTs with GSP, and exploring GTs with GSP as *'such it up and do it'*. While observing their beliefs about teaching GTs with GSP, Cathy appears to have beliefs associated with documenting and manipulating student construction of GTs, engaging students in a variety of activities, and probing them as crucial aspects of assessment. She seems to believe that students' group-work with GSP while teaching GTs can be difficult to engage all of them in a meaningful way with a greater focus and depth of learning. However, she thinks that switching the students' roles in a pair for construction and description of GTs and having conversations with them can be helpful to some extent. However, she laments that partner work may not be ideal with GSP because each student should go through each task on their own to learn in a deeper way. That means she seems skeptical of the effectiveness of group-works by using GSP for the learning of different GTs. Although group work may not bring out good learning to all students, she thinks that discussion between the teacher and students on a one-on-one basis and students' informal proofs through verbal explanation can be productive ways to promote individual activities while teaching-learning of

GTs with GSP. The final category related to exploring GTs with GSP as 'suck it up and do it' points to students' full engagement to more profound learning of GTs with GSP with ownership and responsibility.

On the other hand, Jack seems to believe the formal assessment of students' learning of GTs using GSP with their tasks and activities. He appears to be positive in letting them construct different models with GTs by using GSP and having them explain what it is and how it works. He expresses his belief that students may build different designs of GTs by working together in a small group. Their constructions of GT models may reflect on their creative activity, which can provide evidence of their creativity in using the GSP in GT related tasks. He considers that different groups of students can work with various features of GTs within the dynamic environment of GSP to teach the concepts and procedures to each other. For him, it requires a lot of dynamism of students in the classroom. In this process, students may use the GSP to work on the Ferris wheel or tilted wheel models as a part of individual activities. While dealing with GTs with GSP, for him, time could be an issue because students not only learn the characteristics of GTs, but they also learn about the use of the GSP.

Recommendation

The beliefs associated with actions have implications in the areas of student assessment in a dynamic geometry environment, personal values of individual and group activities while learning GTs, and informal proofs of students through their tasks and activities. The method of assessment can be accommodated with the uses of GSP for teaching geometry in general and GTs in particular. Students' personal beliefs about group activities may affect the classroom dynamics of using the GSP for any geometry lesson in a meaningful way, including teaching GTs. Students' informal proof through verbal explanations and construction activities can be used to promote their thinking and learning of GTs with GSP. When the preservice teachers internalize the use of GSP for teaching-learning of GTs, they may use the tools for other geometry lessons too. Therefore, it is essential to develop a sense of readiness to apply GSP for teaching GTs for the basic understanding of GTs, application of GTs for construction of patterns and models, and creation of different teaching-learning activities to engage students in the dynamic geometry environment of GSP. When preservice and in-service mathematics teachers are engaged in professional development for teaching mathematics with technology (e.g. GSP), they should be fully engaged in the process to develop confidence through enough practice of using technology in the classroom contexts. Hence, training and development of preservice mathematics teachers to use GSP for teaching any geometry concept in general and GTs, in particular, may help them promote a meaningful constructivist teaching approach in their future career as mathematics teachers. Therefore, preservice and inservice mathematics teacher education and development programs should focus on developing positive actional beliefs and dispositions about teaching mathematics with technology in general and teaching GTs using GSP in particular to transform classroom practices. Further, a study is needed to understand the actional beliefs of preservice and inservice mathematics teachers in order to identify how their intended actional beliefs may impact their classroom practices.

Limitations

There are three fundamental limitations in this study— limitations of the RCGT as a theoretical framework, small sample of research participants, and the implicit meaning of participants' voice. The first limitation of the theoretical framework of RCGT is that its assumptions are borrowed from two domains of knowledge—the epistemology of radical constructivism and methodology of constructivist grounded theory. The principle of radical constructivism states that knowledge is actively constructed by the cognizing subjects through the reorganization of their experiences. The function of knowing is associated with the reorganization of the learner's experiential world, but not the memorization of the fact about an ontological reality (von Glasersfeld, 1995). This radical departure of knowledge and ontological reality poses a challenge to integrate the notion of qualitative data and its interpretation through coding, categorizing, and constant comparison in grounded theory (Charmaz, 2006).

The second limitation is associated with a sample size with two secondary level preservice mathematics teachers as a source of data. Although each participant was interviewed five times, making the total number of interviews ten, the data still was limited to two persons as a qualitative case study. In this context, the results of the data in the forms of four categories cannot be generalized to other preservice mathematics teachers of the same level. The third limitation of the study is associated with the explicit and implicit meaning of the participants' voice in the constructed interview excerpts. Their voices in the interview context may not truly represent their permanent actional beliefs about teaching GTs with GSP. When their voices were recorded and transcribed, then there is a loss of a significant amount of information, such as their expressions, body language, and other personal characteristics, which might have influenced their expressions of actional beliefs.

Acknowledgments

We would like to thank Dr. Larry L. Hatfield and Dr. Linda S. Hutchison for their unconditional support to the first author while conducting this study at the University of Wyoming, Laramie, Wyoming, USA. We would like to thank the two anonymous reviewers for their constructive feedback/comments and suggestions on the manuscript.

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