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Development and use of Test Instruments to measure Algebraic Reasoning Based on Cognitive Systems in Marzano's Taxonomy

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Abstract: Algebraic reasoning involves representation, generalization, formalization of patterns and order in all aspects of mathematics. Hence, the focus of algebraic reasoning is on patterns, functions, and the ability to analyze situations with the help of symbols. The purpose of this study was to develop a test instrument to measure students' algebraic reasoning abilities based on cognitive systems in Marzano's taxonomy. The cognitive system in Marzano's taxonomy consists of four levels, including retrieval, comprehension, analysis, and knowledge utilization. According to the stage of cognitive development, students are at the level of knowledge utilization. At this level, students can make decisions, solve problems, generates and test hypotheses, as well as carry out investigations that are in line with indicators of algebraic reasoning abilities. The stages in developing the test instrument were based on three phases: preliminary investigation phase, prototyping phase, and assessment phase. The study obtains a set of valid and reliable algebraic reasoning test instruments for students based on the cognitive system in Marzano's taxonomy. Through the development of an algebraic reasoning test instrument based on Marzano's taxonomy, students can build' thinking habits so that active learning exercises occurs.

Keywords: *Algebraic reasoning, cognitive system, Marzano's taxonomy, Matrix algebra.*

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Introduction

Mathematics learning plays a role in developing student competencies, especially in algebraic thinking and reasoning (Pourdavood et al., 2020). Students can make and evaluate mathematical guesses, identify mathematical patterns and relationships, and justify mathematical thinking and actions through reasoning (Isnani et al., 2020; Otten et al., 2020). Thinking and reasoning refer to the ability to analyze mathematical situations and construct logical arguments. These are habits of thought that can be developed through the application of mathematics in different contexts. It helps students understand what they are learning in mathematics (Wilkinson et al., 2018). However, mathematical reasoning has received less attention in learning mathematics. The emphasis is predominantly on arithmetic skills, performing operations with number and quantities. While there is less attention for mathematical reasoning, focusing on relationships between variables or sets of values (Otten et al., 2020).

Algebraic reasoning is considered a powerful tool for developing mathematical reasoning (Blanton & Kaput, 2011; Cañadas et al., 2016). Algebraic reasoning is the process of carrying out activities to find patterns of mathematical problems or certain contextual situations, make relationships between quantities and make generalizations through formal symbolic representation and manipulation (Kaput & Blanton, 2005). Learning to reason algebra can be referred to as learning to generalize based on examples of mathematical ideas. in addition, algebraic reasoning also learns to construct, justify, and express assumptions about the structure and relationships of mathematics. Teachers can develop algebraic reasoning through problem-solving based on the knowledge and skills students have on algebraic material. Algebraic material is concerned with solving systems of linear equations, determining the value of an unknown, using a system of formulas, equations, symbols, and letters.

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Some students think that algebraic material is a difficult and abstract subject. Even though students are capable in arithmetic, there are still students who have difficulty understanding algebra. Students still rely on memorizing facts and algorithms in problem-solving. Teachers need to make non-routine questions to hone students' reasoning (Basir & Wijayanti, 2020). Marzano's taxonomy can help in constructing a test item to test students' algebraic reasoning abilities. It is because Marzano's Taxonomy has three systems including self, metacognitive and cognitive system (Marzano & Kendall, 2007). Marzano's cognitive taxonomic system consists of four cognitive levels, i.e., retrieval, comprehension, analysis, and knowledge utilization (Colley et al., 2012; Faragher & Huijser, 2014). Therefore, this research aims to develop a test instrument to measure algebraic reasoning based on Marzano's taxonomy of matrix algebra.

Literature Review

Algebra is related to many mathematical concepts (Uygun & Güner, 2019). For this reason, algebraic thinking needs to be developed in school mathematics. the habit of algebraic thinking, a way of thinking that can be obtained in a short time. teachers can contribute to the development of students' thinking in many ways. Driscoll developed habits of algebraic thinking that can be used as a guide in developing algebra, such as Doing/undoing, building rules to represent functions, and abstracting from computations (Eroğlu & Tanişli, 2017).

Algebraic reasoning is more important than procedural skills which tend to be mechanistic (Blech et al., 2019; Bolondi, 2021). It is because the material in mathematics is easier to understand through reasoning. Algebraic reasoning includes all mathematical thinking because of its use in exploring mathematical structures so that it involves various cognitive strategies to understand complex mathematical concepts. Algebraic reasoning also requires students to be able to explore a relationship and build generalization to support a conceptual understanding of the relationship in a formula. Indicators of reasoning in algebraic problems are as follows (Martin, 2009):

a. *Meaningful use of symbols.*

   Identifying existing information; Selecting variables and constructing context in terms of expressions and equations; Interpreting forms of expressions and equations; Manipulating expressions so they can make interesting interpretations.

b. *Connecting algebra with geometry*. Represent geometric-algebraic and algebraic-geometric situations and use connections in solving problems.

c. *Linking expressions and functions*. Using multiple algebraic representations to understand functions and working with function notation.

d. *Mind manipulation*. Linking manipulation with arithmetic operations: Anticipating manipulation results; Choosing a procedure appropriate to the context; Performing Mental Arithmetic.

e. *Reasonable Solution*. Pay attention to the solution steps as a logical understanding of the relationship; find patterns, define rules, and generalize patterns.

Reasoning skills can be trained through mathematical problem solving based on Marzano’s taxonomy. Marzano's Taxonomy moves from a simple way to a more complete process of both information and procedures, less to more awareness of more control over the process of knowledge and how to structure or use it, and a lack of personal involvement or a commitment to a belief and reflection of one's identity. Marzano and Kendall present a taxonomy consisting of two dimensions: the knowledge domain and the levels of mental processing. In the first dimension, there are three distinct domains information, mental-procedures, and psychomotor procedures. There are no hierarchical mental processes. From lowest to highest, these are retrieval processes, comprehension processes, analysis processes, knowledge utilization processes, metacognition processes, and the self-system processes.

Marzano’s taxonomy can be represented as depicted in Figure 1. The rows on the left side of Figure 1 represent three systems of thought and in the case of the cognitive system, four subcomponents of that system. The columns on the right side of Figure 1 depict three different types or domains of knowledge.

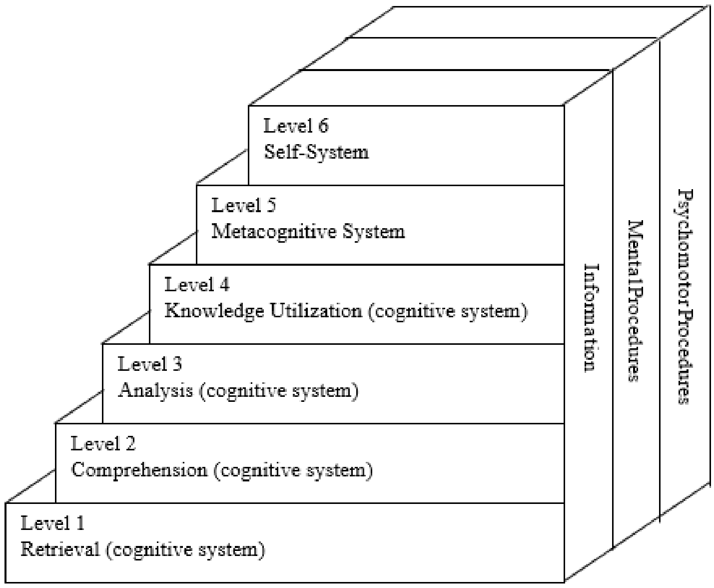

Figure 1. Marzano's Taxonomy

As shown in Figure 1, the second dimension of Marzano's Taxonomy relates to the level of mental processing applied to the three knowledge domains. There are three general systems of mental processing that operate in a coordinated manner: the self-system, the metacognitive system, and the cognitive system. Cognitive systems include four subsystems that have a hierarchical structure: retrieval, comprehension, analysis, and utilization of knowledge (Irvine, 2020). Level 1 *Retrieval* consists of 3 processing categories, including *Recognizing* which means students can validate correct statements about information features but they do not necessarily understand the structure of knowledge or differentiate critical component and criticism; *Recalling* which means students can produce features of information, but they do not necessarily understand the structure of knowledge or distinguish critical and non-critical components; *Executing* which means students can perform procedures without significant errors but do not necessarily understand how and why the work procedure runs. Level 2 *Comprehension* includes information processing components including *Integrating*, in which students can identify the basic structure of information, mental procedures, or psychomotor and critical procedures that are contrary to non-critical characteristics; *Symbolizing* which means students can make an accurate symbolic representation of information, mental procedures, or psychomotor procedures that distinguish critical and uncritical elements (Marzano & Kendall, 2008).

In level 3 Analysis, students can process information through five processes, including *Matching, Classifying, Analyzing Errors, Generalizing, Specifying, Decision Making, Problem Solving, Experimenting, and Investigating*. Matching means that students can identify important similarities and differences from information, mental procedures, or psychomotor. Classifying means that Students can identify superordinate and subordinate categories compared to information, mental procedures, or psychomotor. Analyzing Errors means that Students can identify errors in the presentation or use of information, mental procedures, psychomotor procedures. Generalizing means that students can generalize ideas or new principles based on information, mental procedures, or psychomotor. Specifying means that students can identify the logical consequences of information, mental procedures, psychomotor procedures.

Based on the stage of cognitive development, students are at level 4 Knowledge Utilization. Processing categories at this stage include *Decision Making, Problem Solving, Experimenting, and Investigating*. Decision Making means that students can use information, mental procedures, or psychomotor procedures to make decisions in general or make decisions about the use of information, mental procedures, or psychomotor procedures. Problem Solving means that students can use information, mental procedures, or psychomotor procedures to solve problems in general or solve problems about information, mental procedures, or psychomotor procedures. Experimenting means that students can use information, mental procedures, or psychomotor procedures to generate and test hypotheses in general or generate and test hypotheses about information, mental procedures, or psychomotor procedures. Investigating means that students can use information, mental procedures, or psychomotor procedures to conduct general investigations or conduct investigations about information, mental procedures, or psychomotor procedures.

Methodology

Research Goal

The type of this study is research and development (R&D), which aims to determine the development of a valid algebraic reasoning ability test instrument through content validity and construct validity tests and reliability tests with the help of the Statistical Package for Social Science (SPSS) program. Reliability is the degree of consistency which means that if the test is tested repeatedly, the test provides the same information. In addition, the study aims to describe the use of test instruments as an effort to develop students' algebraic thinking habits. Evaluation criteria related to phases in design research consist of three phases; Preliminary research, Development or prototyping phase, and assessment phase (Plomp & Nieveen, 2013).

Sample and Data Collection

Sources of research data were 28 students of Mathematics Education Department at Sultan Agung Islamic University. The selection of research subjects in quantitative research methods is different from the selection of research subjects in qualitative research methods. The subject selection technique in the quantitative method is based on the cluster type probability technique, with the aim that the research results can be generalized to a broader level.

The data collection technique used a written test of 2 essay questions aimed at measuring the ability of algebraic reasoning on matrix material based on Marzano's taxonomy at the analysis stage and an expert validation assessment questionnaire. Preliminary studies are used to determine initial knowledge as the basis for making prototypes of algebraic reasoning test instruments. Questionnaires were used to validate grids, test instruments, and scoring guidelines for algebraic reasoning tests.

Procedure

The steps of developing an algebraic reasoning ability test instrument in solving matrix algebra problems based on Marzano's taxonomy using a design research type of development study. See figure 2. The emphasis of this type of research is on the development of iterative cycles using formative evaluation. The study used a design consisting of three phases, namely the preliminary research phase, the prototype phase, and the assessment phase (Plomp & Nieveen, 2013).

1) *Preliminary investigation phase*. The researcher conducts an initial investigation by reviewing the relevant literature (curriculum, books, and media) and analyzing the concept of algebraic reasoning.
2) *Prototype phase*. The researcher designs a mathematical reasoning test including a grid, test instrument, and an algebraic reasoning test assessment guide in solving matrix algebra problems based on Marzano's taxonomy.
3) *Assessment phase*. The researcher carried out two activities, construct validation in terms of material, construction, and language by assessing 3 experts and content validation in terms of the validity and reliability of the algebraic reasoning ability test instrument by conducting a limited trial of 28 students.

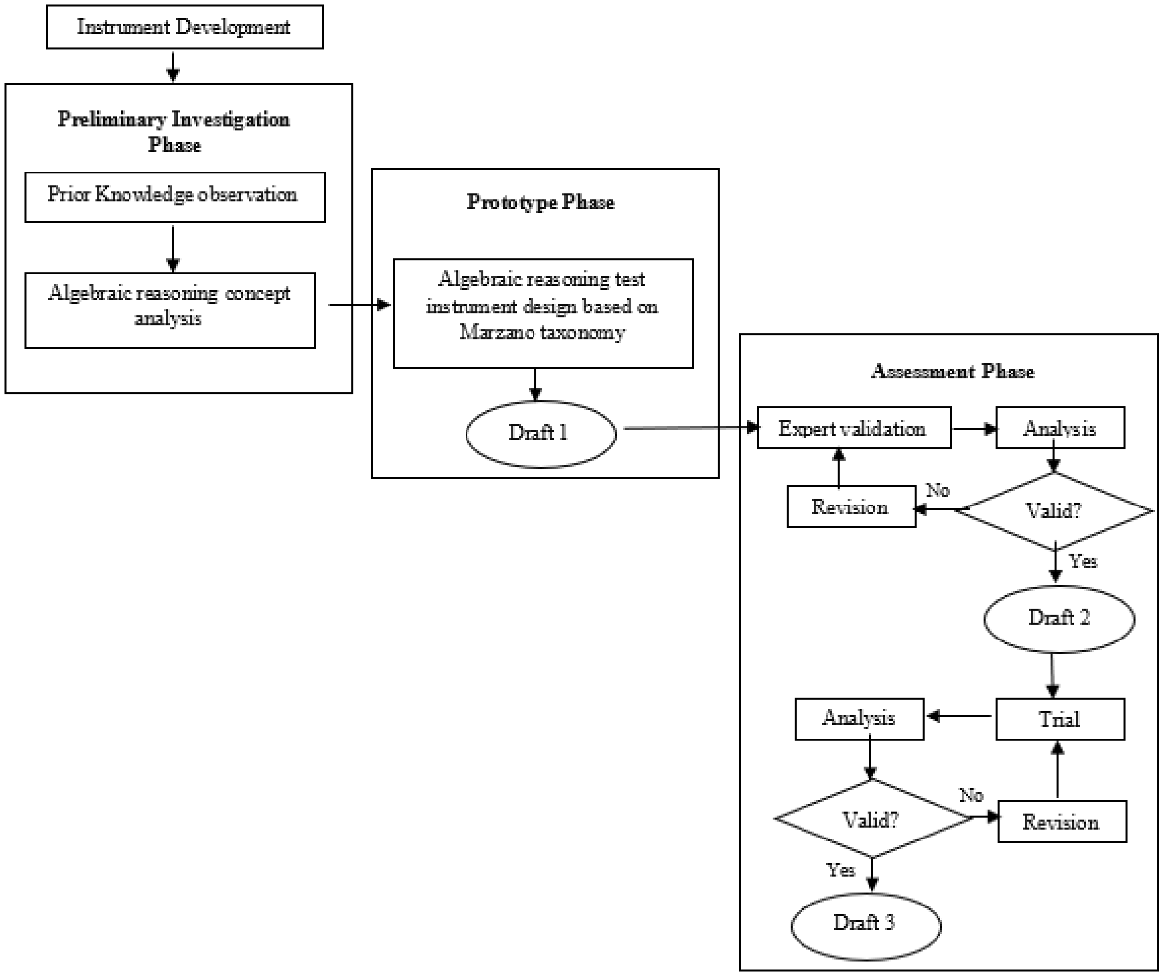

Figure 2. Research Procedure

Analyzing of Data

Quantitative data were obtained from expert validator assessments and analysis of student answers in solving algebraic reasoning test questions based on Marzano's taxonomy. Expert validators come from learning evaluation experts, mathematics education psychologists, and algebra learning experts. Quantitative data analysis is directed to calculate the score of expert validation results (scoring guidelines can be seen in table 1) and determine the validity and reliability of the algebraic reasoning test. The validity of the test items aims to determine how far the test instrument can measure students' algebraic reasoning abilities. Calculation of the validity of the algebraic reasoning ability test item using the help of the Statistical Package for Social Science (SPSS) program. If the item has an r value more than the r at a significance level of 5%, then the item is said to be valid. The reliability test of the items used the Cronbach's Alpha formula with the SPSS program. If the Cronbach's Alpha r_{11} value was more than the r_{table} value at a significance level of 5%, the test instrument was reliable. Or it can be trusted as a good data collection tool.

Table 1. Instrument Validation Score Assessment

Expert Validation Score (EVS)	Remark
$4.00 < EVS \leq 5.00$	Excellent
$3.00 < EVS \leq 4.00$	Good
$2.00 < EVS \leq 3.00$	Sufficient
$1.00 \leq EVS \leq 2.00$	Bad

Qualitative data in the form of suggestions, comments, and input from the validator will be used as the basis for revising the test instrument. The revision covers the material, construction, and language aspects of the algebraic reasoning test instrument based on Marzano's taxonomic cognitive system. After revisions were made according to the input from the validator, the next step was to try out the draft test instrument to research subjects to obtain validity and reliability results.

Findings / Results

The development of a research instrument in the form of an algebraic reasoning test in solving matrix algebra problems based on Marzano's Taxonomy cognitive system used a design research type of development study. The emphasis of this type of research is on the development with an iterative cycle using formative evaluation. The stages carried out consisted of three phases, namely the Preliminary investigation, the prototype, and the assessment phase.

Preliminary Investigation Phase

In the initial investigation phase, an analysis was carried out related to the ongoing matrix algebra learning process. From the results of the analysis carried out, there is a fundamental problem. It is the low results of student learning outcomes in the matrix algebra course. This is because there are still obstacles in learning matrix algebra. The ongoing learning tends to focus on only one direction. It results in student reluctance to take part in the learning process (Abdul & Risqi, 2018). Giving evaluation questions to students is deemed less interpreting the objectives of the expected learning process. The thinking ability and the perceptibility of students cannot be maximally developed (Kusmaryono et al., 2021). Therefore, the researcher is trying to develop an algebraic reasoning test instrument for matrix algebra based on Marzano's Taxonomy cognitive system. The developed instrument is hoped to be used in determining students' mathematical algebraic reasoning abilities.

In this phase, student analysis was also carried out to examine student characteristics that are in accordance with the design and development of the test instrument. Student characteristics include background knowledge and student cognitive development based on Marzano's Taxonomy. In accordance with Marzano's Taxonomic cognitive development stage, students are at the cognitive system of the knowledge utilization level. At this level, students can make decisions, solve problems, generate, and test hypotheses, and carry out investigations that are in line with indicators of algebraic reasoning abilities.

In addition, an analysis of the learning outcomes of the matrix algebra course was also carried out to obtain the degree of suitability of the developed test instruments with the expected learning outcomes as shown in Table 2.

Table 2. Subject Learning Outcomes

Outcomes	Description
Attitude	Demonstrating an attitude of responsibility for work in their field of expertise independently
General Skills	Being able to make decisions appropriately in the context of problem-solving in their field of expertise based on the results of information and data analysis
Knowledge	Mastering the theoretical concepts of mathematics including the concept of matrices in linear equations and transformation of coordinate geometry matrices that support learning mathematics in secondary education as well as for further studies.
Specific Skill	Being able to represent system problems of linear equations using a matrix

The test instrument developed is based on aspects of algebraic reasoning abilities that meet the indicators of meaningful use of symbols, connecting algebra with geometry, linking expressions and functions, mindful manipulation, reasoned solving.

Prototype Phase

The analysis at the initial investigation stage was used to develop an algebraic reasoning test instrument for the algebraic matrix material based on Marzano's Taxonomy cognitive system. The results of the development of the instruments at this stage were called draft 1. The test instrument developed was in the form of essay questions so that the students' algebraic reasoning ability can be known. In developing the instrument, it begins with the activity of arranging the question content outline, the discussion questions, and the scoring guide for each item.

At the stage of preparing the test instruments, the preparation of the items was adjusted to the learning outcomes of graduates. The instrument was developed based on indicators of algebraic reasoning and referred to the development of Marzano's Taxonomy cognitive system as shown in table 3.

Table 3. Test Instrument Grid

Specific Skill	Indicator	Question Number	Aspects of Algebraic reasoning					Marzano Taxonomy Level
			1	2	3	4	5	
Students can represent the problem of a system of linear equations using a matrix.	Determine the appropriate matrix if it is known that the problem is related to a system of linear equations and can find patterns, recognize patterns, and generalize.	1	√	√	√	√	√	Analysis – Generalizing
Students can apply the concept of matrices in solving problems related to matrix transformations in coordinate geometry.	Determine the matrix transformation in coordinate geometry to determine the area and can find patterns, recognize patterns, and generalize	2	√	√	√	√	√	Analysis – Generalizing

Assessment Phase

One of the main criteria to determine whether a research test instrument is used is the result of expert validation. Experts assess the algebraic reasoning test instrument based on material, construct, and language elements. Validation was carried out by three competent people to assess the feasibility of the test instrument consisting of algebra experts, learning evaluation experts, and mathematics learning psychology experts. The preparation of the instrument uses a Likert scale. Each statement item has five kinds of answer choices (1, 2, 3, 4, or 5) which are an assessment of the validity of the algebraic reasoning ability test. The results of the expert validation assessment are shown in table 4.

Table 4. Expert Validator Assessment Results

Assessment Indicators	Expert Validation Score			Average Score	Remark
	E1	E2	E3		
A. Material Learning					
Clarity of instructions for questioning	5	5	5	5.00	Excellent
Suitability of questions with special skills and indicators	5	4	5	4.67	Excellent
The suitability of the questions with the indicators of the algebraic reasoning ability test	4	4	5	4.33	Excellent
The suitability of the content of the material asked for the class level.	4	4	5	4.33	Excellent
B. Construction and Language					
Question items using the correct question word or command.	4	4	4	4.00	Good
Writing questions using the right font size and mathematics equations.	5	5	5	5.00	Excellent
The image display on the question is clear and easy to understand	4	4	4	4.00	Good
The formulation of the questions uses simple language and is easily understood by students.	5	4	4	4.33	Excellent
The formulation of the questions does not cause double interpretation	4	5	4	4.33	Excellent
Formulation of questions using standard language	4	5	5	4.67	Excellent
Average score from expert validation				4.47	Excellent

Table 4 shows that the average score of the expert validation assessment on the algebraic reasoning test instrument is 4.47, which means the test instrument meets the valid criteria in the excellent category. Qualitative data in the form of suggestions and input for the test instrument became the basis for the revision of the draft. The revisions made included increasing the font size in the question instructions, adding the command word explain, choosing the right diction to make it easier for students to understand the questions, adding generalization questions, and using a more structured mathematics equation. The next stage is testing the test instrument given to the research subject.

In the next stage, the test instrument for the algebraic reasoning ability test was carried out on a limited sample of 28 students of the Mathematics Education Department at the Sultan Agung Islamic University. The results of student responses were tested for validity and reliability to determine the feasibility of the questions. An item is said to have validity if the results meet the criteria, in the sense of having parallels between the test results and the criteria (Martin et al., 2020). To find out this, the item validity test was carried out. The technique used to determine the parallelism is the product-moment correlation technique proposed by Pearson. The results of the validity test of algebraic reasoning test items on matrix material based on the Marzano taxonomy cognitive system are presented in Table 5.

Table 5. Validity Results

Item Numbers	The Value of r_{xy}	The value of r_{table}, Sig. 5%	Decision
1	0.844	0.374	Valid
2	0.936	0.374	Valid

Table 5 shows that at the 5% significance level, the value of the r table is 0.374. The first test item has r_{xy} value equal to 0.844. The r_{xy} value is more than the r_{table} value, so the first item is said to be valid. The second item has r_{xy} value equal to 0.936 is more than the r-table, so that the second item is said to be valid. Thus, the algebraic reasoning ability test instrument developed met the valid criteria.

A reliability test was conducted to obtain the level of confidence of the test instrument. The results of the test instrument reliability test used the Cronbach's alpha formula and produced an r_{11} value equal to 0.710. at the 5% significance level, the r_{table} value is equal to 0.374. because the calculation result of r_{11} is more than the r_{table}, the test instrument is reliable. The results of the calculation of reliability using the SPSS program can be shown in table 6.

Table 6. Reliability Test Results

Reliability Statistics	
Cronbach's Alpha	N of Items
.710	2

Thus, the instrument that has been developed fulfills validation, both content validation and construct validation. It means that the algebraic reasoning instrument matrix algebra material based on the Marzano's Taxonomy cognitive system that has been developed can be used to measure students' algebraic reasoning abilities.

Discussion

Algebra is a language for expressing mathematical ideas through symbols known as variables, along with the system that governs the dynamics of these ideas (Bråting & Kilhamn, 2021). Algebraically, numbers are generalized in the form of letters, and relationships that indicate the nature of variables are expressed in the form of equations or inequalities. Algebraic reasoning ability is an ability that focuses on the regularity of the problem-solving process (Obara, 2019).

The behavior and growth of students' cognitive abilities are largely shaped by the classroom environment, including algebra learning activities. One of the things that cannot be separated from algebraic learning is algebraic habits of mind. Students' thinking habits that develop gradually can support them in solving problems in their lives (Eroğlu & Tanişli, 2017).Three main components can familiarize students to think algebraically, including doing/undoing, building rules to represent functions, and abstraction from computation (Driscoll, 1999). The activities carried out to get used to algebraic thinking in detail are described in Figure 3.

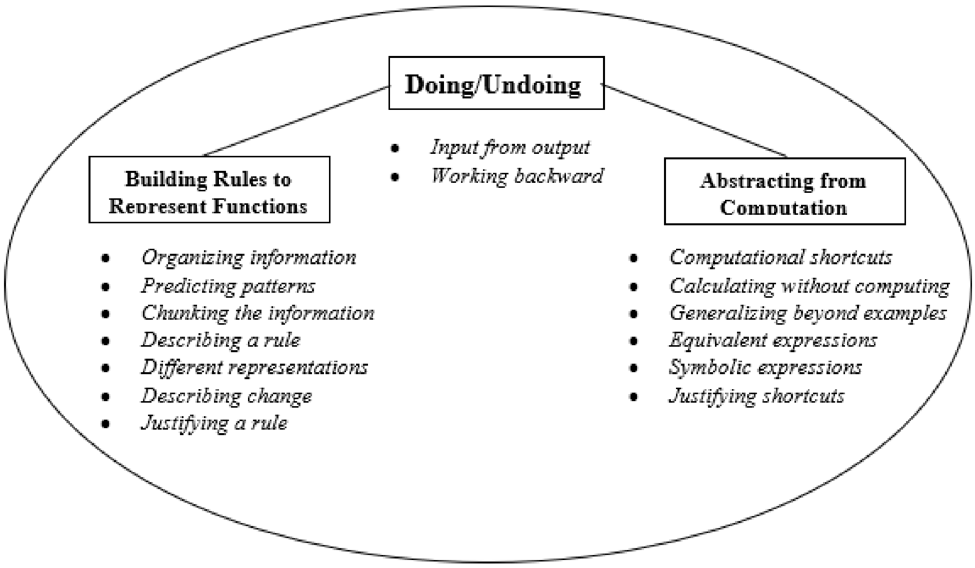

Figure 3. Algebraic Habits of Mind

The following is a description of the aspect of algebraic reasoning abilities based on Marzano's taxonomy at the Analysis – Generalizing level for the first item that has been developed and obtained valid and reliable results. The description in the discussion explains the integration of algebraic thinking habits which consists of five aspects, namely: meaningful use of symbols, connecting algebra with geometry, linking expressions and functions. mindful manipulation, and reasoned solving.

The first question on the algebraic reasoning ability test

Every second, a mixture of 220 ml NO (Nitrogen Monoxide) and 20 ml N_2O_2 (Dinitrogen Dioxide) changes in concentration after heating, i.e., 10% of NO changes to N_2O_2 and 20% of N_2O_2 changes to NO.

a. *Draw a diagram that represents the information!*

b. *Write a matrix equation that represents the information!*

c. *What is the amount of NO and N_2O_2 concentrates after one second, two seconds and 10 seconds?*

d. *What is the amount of NO and N_2O_2 concentrates after n seconds?*

Based on the framework developed by Driscoll, there is a habit of developing algebraic reasoning. In the aspect of *meaningful use of symbols,* students are at the stage of *doing/undoing*. Students can choose variables and build expressions and equations in the context; interpreting the forms of expressions and equations; manipulating expressions so that interesting interpretations can be made. In the end, a mathematical model of the problem to be solved can be obtained. Students can choose the desired variable to assume the objects of the problem presented. They can define these variables in each sentence so that an example or mathematical model can be obtained in the end.

Every second, a mixture of 220 ml NO (Nitrogen Monoxide) and 20 ml N_2O_2 (Dinitrogen Dioxide) changes in concentration after heating

The mathematical model of this information can be written as,

suppose x_i is the number of NO concentrate in milliliters in the i-second, and y_i is the number of N_2O_2 concentrate in milliliters in the x_i-second.

whereas the amount of concentrate before heating can be written as the initial amount of concentrate, x_0 = 220 and y_0 = 20. At every second there is a change, 10% of *NO* changes to N_2O_2 and 20% of N_2O_2 changes to *NO* can be symbolized in the form $x_1 = 0{,}9x_0 + 0{,}2y_0$ and $y_1 = 0{,}1x_0 + 0{,}8y_0$

The objective of the problem is to represent this information in geometry and matrix equations, as well as the concentrations of *NO* and N_2O_2 after one second, two seconds and 10 seconds. In this case, it specifies x_1 and y_1 ; x_2 and y_2; x_{10} and y_{10}. In the next step, it can generalize after n seconds by specifying x_n and y_n.

In the aspect of *connecting algebra with geometry*, students have gotten used to being at the stage of *building rules to represent functions*, where students can represent geometric-algebraic situations and geometric-algebraic situations; using the connection to solve the problem. Students can draw problems in geometric form first to facilitate the structure of problem solving in the questions. The geometric modeling that can be made by students can resemble the following figure 4.

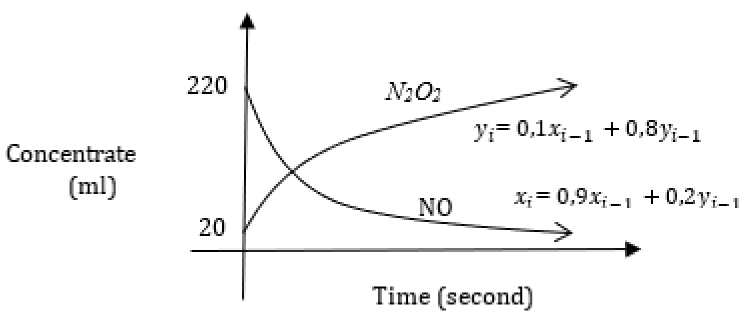

Figure 4. The Geometric Modelling

The geometric form that can be illustrated are not limited as in the picture above. Students can also illustrate with other geometric patterns that represent the problems in that question.

In the aspect of *linking expressions and functions*, students connect expressions and functions using several algebraic representations to understand functions, and work with function notation. In this aspect, they try to connect each function obtained from the question information. They use the matrix as an alternative to find settlement patterns.

$$\begin{pmatrix} x_{i+1} \\ y_{i+1} \end{pmatrix} = \begin{pmatrix} 0{,}8 & 0{,}1 \\ 0{,}2 & 0{,}9 \end{pmatrix} \begin{pmatrix} x_i \\ y_i \end{pmatrix}$$

In the *mindful manipulation* aspect, students get used to doing *abstracting from computation* students relate manipulation to the law of arithmetic; anticipate the results of manipulation; select procedures in context; imagine mental calculations. They perform forms of manipulation that can help solve problems.

- *The amount of NO and N_2O_2 concentrates (in milliliters) after one second can be determined by calculating the multiplication of the matrix by substituting the value $i = 0$, so that it is obtained.*

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 0{,}8 & 0{,}1 \\ 0{,}2 & 0{,}9 \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 0{,}8 & 0{,}1 \\ 0{,}2 & 0{,}9 \end{pmatrix} \begin{pmatrix} 220 \\ 20 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 0{,}8\,(220) + 0{,}1(20) \\ 0{,}2\,(220) + 0{,}9(20) \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 178 \\ 62 \end{pmatrix}$$

Thus, the result of the solution after 1 second is $\begin{pmatrix} 178 \\ 62 \end{pmatrix}$ *which means that the amount of NO concentrate after one second is 178 milliliters and the amount of N_2O_2 concentrate after one second is 62 milliliters.*

- *The amount of NO and N_2O_2 concentrates (in milliliters) after two seconds can be determined by calculating the multiplication of the matrix by substituting the values for $i = 1$, obtaining.*

$$\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0{,}8 & 0{,}1 \\ 0{,}2 & 0{,}9 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0{,}8 & 0{,}1 \\ 0{,}2 & 0{,}9 \end{pmatrix} \begin{pmatrix} 178 \\ 62 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0{,}8\,(178) + 0{,}1(62) \\ 0{,}2\,(178) + 0{,}9(62) \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 148{,}6 \\ 91{,}4 \end{pmatrix}$$

Thus, the result of the solution after 2 seconds is $\begin{pmatrix} 148{,}6 \\ 91{,}4 \end{pmatrix}$ *which means that the amount of NO concentrate after two seconds is 148.6 milliliters and the amount of N_2O_2 concentrate after one second is 91.4 milliliters.*

- *The amount of NO and N_2O_2 concentrates (in milliliters) after 10 seconds can be determined by calculating the multiplication of the matrix by substituting the values for $i = 9$, obtaining.*

$$\begin{pmatrix} x_{10} \\ y_{10} \end{pmatrix} = \begin{pmatrix} 0{,}8 & 0{,}1 \\ 0{,}2 & 0{,}9 \end{pmatrix} \begin{pmatrix} x_9 \\ y_9 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} x_{10} \\ y_{10} \end{pmatrix} = \begin{pmatrix} 0{,}8 & 0{,}1 \\ 0{,}2 & 0{,}9 \end{pmatrix} \begin{pmatrix} 0{,}8 & 0{,}1 \\ 0{,}2 & 0{,}9 \end{pmatrix} \begin{pmatrix} x_8 \\ y_8 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} x_{10} \\ y_{10} \end{pmatrix} = \begin{pmatrix} 0{,}8 & 0{,}1 \\ 0{,}2 & 0{,}9 \end{pmatrix} \begin{pmatrix} 0{,}8 & 0{,}1 \\ 0{,}2 & 0{,}9 \end{pmatrix} \begin{pmatrix} 0{,}8 & 0{,}1 \\ 0{,}2 & 0{,}9 \end{pmatrix} \begin{pmatrix} x_7 \\ y_7 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} x_{10} \\ y_{10} \end{pmatrix} = \underbrace{\begin{pmatrix} 0{,}8 & 0{,}1 \\ 0{,}2 & 0{,}9 \end{pmatrix} \begin{pmatrix} 0{,}8 & 0{,}1 \\ 0{,}2 & 0{,}9 \end{pmatrix} \begin{pmatrix} 0{,}8 & 0{,}1 \\ 0{,}2 & 0{,}9 \end{pmatrix} \cdots \begin{pmatrix} 0{,}8 & 0{,}1 \\ 0{,}2 & 0{,}9 \end{pmatrix}} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} x_{10} \\ y_{10} \end{pmatrix} = \begin{pmatrix} 0{,}8 & 0{,}1 \\ 0{,}2 & 0{,}9 \end{pmatrix}^{10} \begin{pmatrix} 220 \\ 20 \end{pmatrix}$$

Thus, the result of completion after 10 seconds is

$$\begin{pmatrix} 0{,}8 & 0{,}1 \\ 0{,}2 & 0{,}9 \end{pmatrix}^{10} \begin{pmatrix} 220 \\ 20 \end{pmatrix}$$

In the aspect of *reasoned solving*, it notices solution steps as a logical deduction on equality, interpreting the solution in context. Students can find patterns and the characteristics of algebraic arithmetic operations from mathematical problems and draw conclusions from the questions given. The process of drawing conclusions about algebraic reasoning problems is through the processes of finding patterns, recognizing patterns, and generalizing the patterns found.

Students try to find patterns in the following process. Patterns are found through processing information from questions. This information is used to help solve the problem of many concentrations of NO and N_2O_2 (in milliliter) after n seconds which can be determined by describing the multiplication of the matrix by substituting the value $i = n - 1$, obtaining.

$$\begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{pmatrix} 0,8 & 0,1 \\ 0,2 & 0,9 \end{pmatrix} \begin{pmatrix} x_{n-1} \\ y_{n-1} \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{pmatrix} 0,8 & 0,1 \\ 0,2 & 0,9 \end{pmatrix} \begin{pmatrix} 0,8 & 0,1 \\ 0,2 & 0,9 \end{pmatrix} \begin{pmatrix} x_{n-2} \\ y_{n-2} \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{pmatrix} 0,8 & 0,1 \\ 0,2 & 0,9 \end{pmatrix} \begin{pmatrix} 0,8 & 0,1 \\ 0,2 & 0,9 \end{pmatrix} \begin{pmatrix} 0,8 & 0,1 \\ 0,2 & 0,9 \end{pmatrix} \begin{pmatrix} x_{n-3} \\ y_{n-3} \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} x_n \\ y_n \end{pmatrix} = \underbrace{\begin{pmatrix} 0,8 & 0,1 \\ 0,2 & 0,9 \end{pmatrix} \begin{pmatrix} 0,8 & 0,1 \\ 0,2 & 0,9 \end{pmatrix} \begin{pmatrix} 0,8 & 0,1 \\ 0,2 & 0,9 \end{pmatrix} \cdots \begin{pmatrix} 0,8 & 0,1 \\ 0,2 & 0,9 \end{pmatrix}} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{pmatrix} 0,8 & 0,1 \\ 0,2 & 0,9 \end{pmatrix}^n \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

Thus, the calculation after 10 seconds obtains.

$$\begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{pmatrix} 0,8 & 0,1 \\ 0,2 & 0,9 \end{pmatrix}^n \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

After finding the correct and valid pattern, students can recognize the pattern that can be used as a reference at the pattern generalization stage. In the end, they can generalize such patterns to answer the problems in the provided questions.

Thus, algebraic reasoning can be developed in various ways, starting from developing arithmetic generalizations, developing thinking a variety of ways, to practicing The Algebraic Habits of Mind. The most important thing that teachers must pay attention to in algebra learning is to provide open-ended problems and scaffolding that can improve students' algebraic reasoning abilities.

Conclusion

The algebraic reasoning test instrument is based on the Marzano taxonomy cognitive system on matrix algebra with an average expert validation score of 4.47 or 89.4% with excellent criteria, the validity test of the two questions is 0.844 and 0.936, which means the test instrument has high criteria and a reliability level of 0.749 which indicates the criteria are feasible. Thus, the results of this research and development instrument can be used as an identification tool and as a measuring tool for students' algebraic reasoning abilities, especially in matrix algebra material.

Recommendations

Based on the research results of the algebraic reasoning ability test instrument based on Marzano's taxonomy on a valid cognitive system, it is necessary to disseminate it to determine its practicality and effectiveness. Research can also be developed by making test instruments based on other taxonomies. In addition, it is also possible to study the cognitive load of students in solving algebraic reasoning problems. Identification of students' cognitive structures in solving algebraic problems is also very important for advanced studies.

Limitations

This research is a research and development of an algebraic reasoning test instrument based on Marzano's Taxonomy on matrix material so that it is possible in other respects to develop a test instrument on other learning materials. Researchers who are interested in carrying out similar research can apply it to different materials.

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Author Contribution Statement

Basir: Concept and design, drafting manuscript, writing. Waluya: Supervision. Dwijanto: Critical revision of manuscript. Isnarto: Data acquisition and data analysis

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