The Observed Impact – Implementing Inquiry: Based Learning at a Calculus Class

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Abstract: This study investigated how implementing inquiry-based learning (IBL) can be an effective tool for an instructor to conduct rich formative assessment. Many researchers have documented that IBL promotes active learning from students' learning perspective. However, little research examines how IBL affects instructors' teaching practice from teaching perspective. Based on the data collected from a Calculus II class, the author discussed how the structure of IBL class produced rigorous on-going formative assessment during classroom teaching from the three aspects: helping the instructor "see" student thinking; helping the instructor "see" the level of student understanding; helping the instructor catch teachable moments. The rigorous on-going formative assessment, in turn, helped change student classroom behaviors in terms of asking more questions, showing deep thinking, and gaining confidence.

Keywords: Inquiry-based learning, on-going formative assessment, structure of a lesson, pre-class assignment.

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Introduction

Formative assessment plays a very important role for students to achieve positive learning outcomes (Bransford et al., 2000; Darling-Hammond, et al., 2008; Hattie, 2009; Heritage, 2010). In order to improve student learning, formative assessment must be a part of the learning cycle for students to continuously receive just in-time feedback (Friesen & Scott, 2013). There are many ways to collect data for analyzing student learning, to name a few, written student work, class survey, oral data, and on-line collected data (Black & Harrison, 2004; Ruiz-Primo & Furtak, 2006; Smit & Birri, 2014). In the literature, research on assessment has been well established, specifically on summative assessment (Rönebeck et al., 2013). However, research on concrete formative assessment still needs more further investigation. Although many theoretic strategies were proposed, there are questions which remain unanswered. This paper focus on ongoing formative assessment during classroom teaching in an inquired-based learning (IBL) environment. Based on the data collected from a Calculus II class taught by the author, the paper intends to answer the two questions: 1. To what extent can the structure of a class help increase opportunities for on-going formative assessment during class teaching? 2. To what extent can active learning model change student behavior in mathematics classroom?

The Framework of Teaching Design

Different teaching design leads to different learning outcomes. Many well-documented studies suggested that inquiry-based learning (IBL) promoted learning from a variety of perspectives including facilitating deep learning, motivating self-directed learning, promoting high-order of thinking, and reinforcing collaborative learning (Sockalingam et al., 2011; Hwang & Chang, 2011; Marks, 2013; Gu et al., 2015; Guido, 2017; Gholam, 2019). IBL engages students in the process of learning and deepens their understanding of mathematical ideas/concepts by doing mathematics in class. Research in the international literature agreed that IBL is a teaching approach that facilitates deep learning. What is deep learning referred to? Sawyer (2006) described deep learning as follows:

- Learners relate new ideas and concepts to previous knowledge and experience.

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Learners integrate their knowledge into interrelated conceptual systems. 
learners look for patterns and underlying principles.
learners evaluate new ideas and relate them to conclusions.
learners understand the process of dialogue through which knowledge is created and 
can examine the logic of an argument critically.
learners reflect on their own understanding and their own process of learning. 

In mathematics classroom, active learning model promotes deep learning, produces effective teaching/learning, provides opportunities for students to claim their learning ownership (Liang, 2018). Capturing all the characteristics of an active learning model, an IBL classroom facilitates meaningful mathematical discourse. In IBL classroom, a teacher motivates students to share their understanding of mathematics by engaging students in tasks that involves mathematical reasoning and problem-solving, and students involve plenty of discussions in class. Strategies of classroom discourse are crucial for IBL teaching approach. When implementing IBL in my classroom, I utilized the Five Practices for Effective Discourse:

- Anticipating student responses prior to the lesson.
- Monitoring students' work on and engagement with the tasks.
- Selecting particular students to present their mathematical work.
- Sequencing students' responses in a specific order for discussion.
- Connecting different students' responses and connecting the responses to key mathematical ideas.

Guided by the Five Practices, I developed/design my course activities (Smith & Stein, 2011).

Methodology

This study utilized qualitative research method to investigate how implementation of IBL in a calculus class have ongoing impact on teaching and learning from the aspect of formative assessment. The research follows a qualitative research scheme. The research setting is a Hispanic-serving university with semester system. There were 46 students enrolled in the Calculus II class. The class met twice a week and each lesson lasted 100 minutes. We will describe structure of teaching design, data collection, and analysis as follows.

Structure of Teaching Design

Before each lesson, a pre-class assignment was assigned and posted on BlackBoard for students to complete in order to get them prepared for in-class discussions. Pre-class assignments carried 5% of the course grade and must be turned in on BlackBoard. Here is a sample pre-class assignment below:

**Week 3 – 8.3&8.4 (Due: 1/28, 4:00PM)**

1. Reflecting on what we did for 8.2 on Wednesday class, what did you learn from doing the problems? Any questions are still puzzling you? Be specific. Any confusions you had before and then you get yourself cleared and understood someway, please explain If you have any to share (This is a description of product failure, see our syllabus for detail).

2. List all the tools you have now in your tool box to solve problems of integration by parts.

3. Study the examples in 8.3 (Page 475-479), try to solve three of them on your own. What strategies have you noticed through solving the problems?

4. Study the examples in 8.4 (Page 481-483), try to solve two of them on your own. What strategies have you noticed through solving the problems?

Each lesson was structured as shown in Table 1:

<table>
<thead>
<tr>
<th>Allotment Time</th>
<th>Activities</th>
<th>Who</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 (minutes)</td>
<td>Scaffolding Discussion</td>
<td>Whole Class</td>
</tr>
<tr>
<td>60 (minutes)</td>
<td>Doing Mathematics (Problem-Solving, Student Presenting/Sharing/Discussing)</td>
<td>Individual/Groups/Whole Class</td>
</tr>
<tr>
<td>20 (minutes)</td>
<td>Reflecting on the Mathematics Just Done</td>
<td>Whole Class</td>
</tr>
</tbody>
</table>
Scaffolding Discussion.

The first 20 minutes of scaffolding discussion include two components: 1. the big idea/main concept that was learned from previous class and the questions/confusions if any. 2. The new concepts and their applications that is going to be learned in the current class. During the scaffolding discussions, students would share their reflective thinking of the main ideas investigated last class, including asking any related questions or clarifying confusions; after reflecting on previous class content, students then would discuss their understanding of new idea/concept/application that will be grasped in the current class based on pre-class homework questions. The pre-class homework was designed to get students prepared for new knowledge explored in class. Students must complete a pre-class homework and turn their completed work on BlackBoard before each lesson. Since pre-class homework is an integral part of learning, the completion of pre-class homework is counted toward 5% of the course grade. The pre-class homework is not only for students to pre-study the new content being learning in class but also for the instructor to gain formative data of student understanding and some common misconception/mistake in order to address the identified problems during the class discussions.

Doing Mathematics.

About 60% of lesson time is assigned for students to actively doing mathematics in terms of problem-solving, questioning, presenting their solutions for the problems assigned in class, sharing their ideas with peers, and discussing some disagreed/confused points for better understanding. During this period of time, very rich informative assessment opportunities were presented to the instructor. The instructor could see knowledge gaps, identify misconceptions, and observe some common mistakes. Immediately addressing the problems observed would prevent students from accumulating knowledge gaps and misconceptions.

Reflecting on the Mathematics Just Done.

For the last 20 minutes of a lesson, students were given an opportunity to reflect on what they have just learned through doing mathematics. A summary of main mathematics idea(s) for the lesson would help students internalize new ideas, relate what just learned to previous knowledge, and retain the newly gained knowledge. This conclusion part of the lesson also helps the instructor see the level of student understanding through students’ reflective oral responses.

Data Collection and Analysis

Data Collection included instructor’s observations, student work, student oral reasonings, midterm surveys, post surveys, student reflections, instructor’s reflections, student course evaluations. Qualitative inductive method was utilized to analyze the data. Guided by the research questions, the author started with reviewing the collected data and looking for emerging themes. Since data triangulation can provide a more detailed and completed picture of the situation (Altrichter et al., 2008), repeated comparisons were conducted to validate the findings. Since the instructional triangle is composed of the three factors – mathematical tasks, teaching, and students (Cai et al., 2020; Chhen et al., 2003), the analysis was conducted through the systematic verification process in the instructional triangle. For example, the author found that the iBL approach constantly created opportunities for the instructor to gain on-going formative assessment through their completed pre-class assignments and in-class activities such as students writing process of reaching a solution, student orally presenting their thinking, group discussions, questions asked when solving a problem. The finding was confirmed by student midterm surveys, student reflections, and course evaluations. Comparison matrices were utilized to look for the converged findings. Here is a sample matrix:
<table>
<thead>
<tr>
<th>Converged Theme</th>
<th>Instructor: Observations &amp; Reflections</th>
<th>Students’ Post-Surveys &amp; Reflections</th>
<th>Course Evaluations</th>
</tr>
</thead>
</table>
| Pre-class Assignments - a positive impact on learning | Pre-class assignments provided opportunities for students to engage in self-regulated learning | 97% (34/35) of students who responded the surveys indicated that Pre-class assignments benefited their learning. Typical Responses:  
• Really do help me learn the material better  
• Helped me understand what is going on in class a lot better  
• Encouraged me to look though material before a class and helped me to have a much clearer understanding during further explanation of the concepts  
• Made me actually read the textbook, making learning easier, giving me an idea of what is happening in class | The course evaluation results indicated that for the item “The instructor encouraged me to take an active role in my own learning, 81.3% of the students marked a 5 and 12.5% of the students marked a 4 and 6.3% of the students marked a 3 providing a Likert scale of 1 to 5 (5 - Strongly Agree; 1 – Strongly Disagreed) |

| Student Engagement in learning -increased           | Students took more active role in learning through doing Pre-class assignments and engaging in class discussions | In the post-surveys, responding to their take-aways from the course, students realized how engagement affected their learning in addition to mastering course content. Sample student responses:  
• “Inquiry-based learning” allowed me to interact with others in class and clarify questions I had  
• I felt that I learned a lot more than last semester. This class really retained my reading and learning  
• Always study and master the material before going to class; never do assignments later than due time  
• The bulk of learning is from doing practice problems  
• I improved my knowledge in calculus and studying habits | 94% of the students agreed that “The instructor encouraged me to take an active role in my own learning”. |

As demonstrated in the table 2, the converged theme - Pre-class Assignments had a positive impact on learning and increased student learning engagement, was emerged from instructor’s Observations & Reflections, students’ post-surveys & reflections, and course evaluations.

**Findings/ Results**

Data analysis was guided by the two research questions: 1. To what extent can the structure of a class help increase opportunities for on-going formative assessment during class teaching? 2. To what extent can active learning model change student behavior in mathematics classroom? Reviewing repeatedly the data including instructor’s observations, student work, student oral reasonings, midterm surveys, post surveys, student reflections, instructor’s reflections, student course evaluations, I went through the process of initial coding, comparing the emerging themes from the different data source, and generating the findings that are convergent (see the sample matrix in the section of methodology). Analysis of the collected data revealed that the structure of an IBL class shaped student learning experience for the better through extensively engaging students in learning and constantly created opportunities for the instructor to gain on-going formative assessment and to be able to immediately address the learning issues during classroom teaching. The emerging themes generated from the data includes:
• The structure of IBL class shapes the way how the formative assessment is conducted by the instructor.
• IBL as an active learning model positively changes student classroom learning behaviors.
• The pre-class assignments have a positive impact on student learning.

In next section, the findings will be discussed in detail.

Discussion

Findings in the international research literature indicated that IBL helped students learn more, understand deeper and improve academic achievement in reading, mathematics and science study (Newmann et al., 1996; Smith et al., 2001; Sawyer, 2006; Barron & Darling-Hammond, 2008; Hattie, 2009; Scott & Abbott, 2012). The learning gains through IBL approach is closely related to formative assessment (Bransford et al. 2000; Darling-Hammond et al., 2008; Hattie, 2009; Heritage, 2010). Friesen and Scott (2013) promoted the idea that formative assessment is a necessary component of a learning cycle because it provides students ongoing feedback for improving their work and understanding. Koksalan and Ogan-Bekiroglu (2020) proposed the integration of formative assessment and IBL approach. This study consents to the research findings in the literature and explores further that the structure of IBL class makes it take place naturally for instructor and students to benefit from the ongoing formative assessment in the cycle of learning. In this section, we will discuss in detail the three findings engendered by the study: How did the structure of IBL class shape the way of the formative assessment conducted by the instructor? How did IBL as an active learning model positively change student classroom behaviors? How had the pre-class assignments positively affected student learning?

The Structure of IBL class shapes the way of formative assessment

How a class is structured determines when and how formative assessment takes place. In a lecture class, an instructor talks most of time during a class. Teaching mathematics at college level for many years, I had taught by lecturing most of my teaching career. I had always had the issue about on-going formative assessment during class teaching time. When I asked questions, either only a few students responded or all of them kept silence. I had been busy with talking most of class time, hardly knew student thinking, and had fewer opportunities to catch student misconceptions. The scenario took place in every lecturing class and my colleagues had the similar experience as mine. The structure of a lecture mathematics class makes an instructor on the center stage of doing mathematics and put students in the background listening inactively. There is very limited informative assessment taking place during class time. On the contrary, the structure of an IBL class puts students on the center stage doing mathematics and provides many opportunities for a teacher to observe the process of students’ conducting learning. In the Calculus II class, the formative assessment was on-going, starting from Pre-class assignments throughout every class period. Before each class, reviewing the completed Pre-class assignment provided the information for the instructor to see some key learning issues needs to be addressed during the class discussion time. During each class period, either observing students’ working a problem or listening to student discussions and questions constantly informed the instructor of students’ understanding or stuck points which helped her decide how to conduct a just-in-time intervention in order to support student learning. There were countless formative assessments happening from reviewing PA assignments to conducting class activities in classroom. I would like to share several examples in the following.

• In week 5 Pre-class assignment, a student wrote: "I do not understand how to determine the shell height or radius. Once you determine the height and radius you can just take integral of the function and plug in the upper and lower just as in the disc method", I was informed that we needed to discuss in detail how to find a shell height and radius when using shell method in order to help students understand better.

• During a class time, I observed that many students didn’t know how to calculate a number with fraction power. When calculating the length of the curve which is the graph of function

\[
y = \frac{4\sqrt{2}}{3}x^{3/2} - 1, \quad 0 \leq x \leq 1
\]

\[
\frac{dy}{dx} = \frac{4\sqrt{2}}{3}, \quad \frac{3}{2}x^{1/2} = 2\sqrt{2}x^{1/2}
\]

\[
L = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx = \int_0^1 \sqrt{1 + 8x} \, dx
\]

\[
= \frac{2}{3} \left[ \frac{1}{8} (1 + 8x)^{3/2} \right]_0^1
\]

\[
= \frac{1}{12} \left[ (1 + 8 \cdot 0) \frac{3}{2} - (1 + 8 \cdot 0) \frac{3}{2} \right]
\]
Students got stuck here.

- During a class time, I noticed that many students didn't know how to solve the problem:

\[ \int (\sqrt{x} \cdot \sqrt{1 + \frac{1}{x}}) \, dx \]

because they didn't know how to simplify \( \sqrt{x} \cdot \sqrt{1 + \frac{1}{x}} \).

- During a class time, a common mistake appeared when calculating the length of the curve

\[ y = \frac{1}{2} (e^x + e^{-x}) \text{ from } x = 0 \text{ to } x = 2 \]

\[ L = \int_0^2 \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx = \int_0^2 \sqrt{1 + \left( \frac{1}{2} (e^x - e^{-x}) \right)^2} \]

\[ = \int_0^2 \sqrt{1 + \frac{1}{4} (e^{2x} - e^{-2x})} \]

(Common mistake: \( (e^x - e^{-x})^2 = e^{2x} - e^{-2x} \))

Very commonly students had the knowledge gap of algebraic work to be addressed when solving a calculus problem. Students often made mistakes such as \((2x + 1)^2 = 4x^2 + 1 + \frac{2x^2}{2x^2 + 4} = \frac{1}{4} e^x \cdot e^x = e^{2x}\). The missing knowledge of algebra hinders student learning by misleading their thinking and understanding. Just-in-time interventions from the instructor will help students close the gaps and move forward in the path of learning.

**IBL positively changed student classroom behaviors**

Student classroom behaviors significantly affect learning atmosphere. In the beginning of the semester, most of the students were shy of sharing their thinking during class discussions. I constantly emphasized that mistakes were very important stepping stones for learning and always highly valued the students who contributed their thinking to the discussions, specifically at times when a mathematical mistake was made. I used the mistakes as examples for the students to see how they helped them learn. Gradually, more and more students felt comfortable to ask questions and were not afraid of making mistakes in front of class. I noticed that even a few very quiet students approached me to ask questions or shared their solutions on chalkboard. There was a student who usually sat in a corner of the classroom and was not active in class discussions for the first few weeks. When I stopped by to look at his work at his desk, he started to ask me very thoughtful questions and I was not shy of telling him so with encouragement. When we discussed finding surface area of revolution, he shared his thinking to the whole class and at the end he said: “Yes, I am a genius, just kidding” which made the class laugh.

On-going classroom formative assessment provided immediate feedback to the students in the Calculus II class. Misconceptions/mistakes/knowledge gaps were addressed without delay. When a learning hurdle was removed, the students felt good about themselves. There were many times I heard students saying: “wow, I did it” or “I got it” with joyful voice when having a problem completely solved after understanding what made him/her stuck. I saw positive impacts on student behavior in class. Students also noticed some changes taking place in their learning. Here are some comments from students in the surveys or the course evaluations:

- Doing problems on the board and seeing common mistakes that I get caught up on and having other perspectives velar confusing ideas along with me.

- I am much more confident in my process after being scrutinized while solving problems. I am now more aware of small details that I might’ve missed otherwise.

- I understand how to better learn a topic on my own time and efficiently, using the right techniques.

- I noticed I have become more independent when trying to solve problems.

- I am able to ask a lot of questions. I notice that I’m more dedicated to understand.

- This class improved my study habits.

- I find myself spending more time on practice problems to learn the material.

- I put so many hours in this class.
The Pre-class assignments had a positive impact on student learning

Although my research questions did not include how Pre-class assignments affect student learning in an IBL classroom, analysis of the data generated the finding that the Pre-class assignments had been recognized by the students as a positive factor for their learning. In either the midterm survey or the post-surveys, students indicated that doing the Pre-class assignments helped them understand the discussed materials better in class. One student wrote: It makes me really actually read the textbook, making learning easier, giving me an idea of what is happening in class. Another student commented: It help a lot in understanding the base concept of the material that was going to be taught that day. Pre-class assignments made students start a new routine – reading the textbook before each class because they had to consult the textbook to answer the questions in Pre-class assignments. Nowadays reading mathematics textbook is not common practice for our college students. Majority of the students in the class appreciated how the Pre-class assignments helped their learning although some of them suggested that they should be rewarded more than 5% credits for the Pre-class assignments. In the post-surveys, one student stated that I greatly appreciated that the PA assignments “having an idea of what to do before class was paramount to my success”. Another student wrote: “This class really refined my reading and learning”.

Conclusion

The structure of a lesson helps shape how students learn and how an instructor teaches through creating a dynamic learning environment. The research of implementation IBL in Calculus II class showed us the evidence that IBL – an active learning model had positive impacts on both teaching and learning. From teaching perspective, lessons structured by IBL produced rigorous on-going formative assessment during class teaching from three aspects: helping the instructor “see” student thinking; helping the instructor “see” the level of student understanding; helping the instructor catch teachable moments. From the learning perspective, the rigorous on-going formative assessment, in turn, helped change student classroom behaviors in terms of asking more questions, showing deep thinking, and gaining confidence. The actively engaging class environment provided a dynamic that make students want to be a part of the learning community. Students were brought up to the center of the stage from the background while the teacher retreated to the background, listening, observing, guiding, and conducting just-in-time intervention when needed. Additionally, practicing IBL in class promoted diversity and equality by inviting each student' participation and embracing inclusiveness.

Recommendations

This study is based on the data collected from one calculus II class for one semester. Research should be extended to more calculus classes for more semesters. As an instructor, I observed that student engagement in the IBL class was significantly higher than that in my previous calculus classes which were taught in a traditional way; I noticed that students passing rate in the calculus II class was higher than that in my other calculus classes of previous semesters; I also observed that the structure of IBL class promoted equality by including every student in learning in a diversity classroom. Extensive study will be needed to validate these observed phenomena. In addition, since some students in the course survey saying that the class changed their study habits, I have the following questions to be answered in the future research: To what extent does IBL approach affect the formation of students’ study habits? Can IBL approach help students form a growth mindset?

Limitations

This research is limited to a calculus II class for one semester. The observed phenomenon will be good starting points for further study in the future. The shared result is a preliminary report that cannot be generalized to all calculus classes in other settings. More calculus classes for more semesters are needed for an extensive study.

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