An On-Campus Approach to Online Mathematics Teaching: A Case Study on a Pre-Calculus Course

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Abstract: Learning in STEM subjects is to a high degree based upon understanding logic, especially in subjects like mathematics. It has always been challenging to preserve the benefits of on-campus teaching and learning while digitalizing the teaching of mathematics. In this article an approach to design for a suitable online pre-calculus course is discussed, that aims to address the challenges. The main focus will be on student active learning in synchronous online environments, technical teaching methods in lectures, and pre-planning of the course. The final exam in the course was held as a closed-book proctored exam on-campus with pen and paper, providing data on comparisons of the final exam scores with the exam from the previous year, in which the entire course was held on-campus. The results indicate a positive effect from the presented design. Also, student surveys indicated high student satisfaction.

Keywords: Face-to-face, online teaching, student active learning, STEM teaching, technical teaching methods.

Introduction

The quality of both the technology and the methods of distance learning in each subject has become an important topic in today’s higher education, boosted by the COVID-19 lockdowns all over the world. The change from traditional classroom teaching where a teacher is available in the same space and at the same time towards distance education, where the teacher does not have to be at the same time in the same digital room has brought some challenges.

Several research papers written during the period of COVID-19 showed that many of the problems are due to the technical issues such as unstable networks and lack of sufficient digital and technical skills among teachers and students (e.g., Hassan & Hussain, 2020). Technical issues have several sites as due to a lack of accessible digital tools, technology or experience; it is more difficult for the teacher to follow up with students and provide guidance on the taught topic to the same extent and quality as on-campus. However, from the pedagogical and social point of view, the lack of face-to-face interaction and communication was considered a negative side of distance education both before and during COVID-19 (Hassan & Hussain, 2020; Mahlangu, 2018; Muilenburg & Berge, 2005; Todd, 2020). Todd (2020) has cited some of the teacher comments about the pedagogical and social issues when having online teaching “not really seeing each other makes it difficult to predict the emotional situation” and “if we were studying in the classroom, we would be able to walk to the students to see what they are doing, to ask, to see what they have written, but online, we cannot do anything”.

These citations point to something that the authors have felt and experienced as well in their own teaching. The authors have felt that online courses that are based upon asynchrnous learning methods, like static videos and quizzes, provide less degree of interaction between student and teacher or between fellow students. Without access to a physical learning environment, many students are easily distracted and lose focus on their work (Bringula et al., 2021; Hadi Mogavi et al., 2021), which can decrease learning outcomes and/ or makes dropping out of the studies easier.
Several other studies discuss the same concern (e.g., Bognár et al., 2018). The delivery of synchronous online education has increased significantly during and in the wake of COVID-19. Further studies and explorations of the potential and opportunities of this educational approach are therefore needed.

One of the main benefits of on-campus teaching is that the teacher can use non-verbal social cues like body gestures and eye contact to engage students during the class, which can be challenging to achieve in online teaching. Another challenge related to online teaching is to make the teaching environment and situation genuinely fun and engaging. Sharing one’s own thoughts might be easier in a physical environment when the other person’s body language and facial expressions can be seen, and one can adjust to it quickly. All these belong to the five themes (stimulating pedagogy; lecturer engagement; a safe learning space; shared experience; and a low-stress environment) that encapsulate what higher education students perceive to be a fun learning experience (Whitton & Langan, 2018). They are also examples of social presence, teaching presence, and cognitive presence that was found to be the key elements in successful distance education (Maddrell et al., 2017).

In this paper, the preparation of an online synchronous pre-calculus course that aims to address the pedagogical and social challenges of online education is discussed. Even though the technical issues are vital to be solved, in this paper the authors do not consider the possible technical issues but assume that the students have necessary network, digital devices, and programs available, the first provided by the students themselves and the latter by their educational institution.

The main focus of the paper is to show how the benefits of the classroom teaching environment (in mathematics) can be achieved in online teaching by using the current technical teaching tools and methods. The success of the online course design and completion is measured by comparing the course’s exam results to the previous year’s on-campus course exam results.

**Literature Review**

The first step towards achieving the goal of this study was to identify the elements in a classroom environment that made teaching fun, efficient, including, and stimulating, but were also challenging elements to achieve in online education. These elements are identified by the authors and colleagues during their long classroom teaching experience and while turning the classes into online teaching. Also, a literature study to cross reference the initial elements to existing research findings has been carried out. The identified elements, discussed in this section, are then addressed on the pre-calculus online course given in the summer 2020 at the Norwegian University of Science and Technology (NTNU).

**Importance of Preparation Phase**

Clear instructions are a very important part of a success in online courses (Albelbisi et al., 2018). The need for instructions starts well before the course starts and reaching all students with the necessary information has always been a challenge, especially with fresh students. For instance, problems can occur for students when trying to access important course information on learning platforms which require university student accounts. The problems can both be related to acquiring the student accounts and to logging on properly. The challenge is even greater if the course is held online at the beginning.

When the communication to a large degree relies upon apps and technologies, with a large variance of technological knowledge and equipment among the students, one technical failure can cause the student to not acquire the necessary information when both the lectures and exercise classes are held online. The same has been discussed in the study of (Bognár et al., 2018) when designing virtual learning environments for calculus courses. It is crucial to always have several back-up plans prepared when the students need help on short notice.

In order to help and guide the students in an optimal way, it is important that both the teacher and tutors are well-prepared before the course (Albelbisi et al., 2018; Bognár et al., 2018; Lopes & Soares, 2018; Trenholm et al., 2019) and that they find an efficient and convenient way to communicate and collaborate. Both teachers and tutors should have the necessary knowledge and practice on how to organize during the course.

**Important Elements of Lectures in Classroom Environment**

Albelbisi et al. (2018) mapped the success factors for Massive Open Online Courses (MOOC) in higher education. The factors that are related to lectures were interactivity between the instructor and the students, and competent and available instructors. They also mentioned that short videos where the instructor talks and presents information with drawing-hand style are a useful factor for student enhancement.

Being physically present in a class with the possibility to interact with the teacher and fellow students face-to-face is beneficial for the students’ motivation and engagement (Fiorella et al., 2019). The students will more easily be able to establish a social connection with the teacher, which will help concentration during the lectures (Stull et al., 2018). In
addition, the teacher will be able to interpret how the students perceive the taught subject by watching their facial expressions.

These above-mentioned factors: interactivity, physically present instructor, and hand-drawn examples, are those that are easily achievable in a classroom environment and can be divided into smaller elements. Environments that include interactivity and available teachers mean actions such as bodily interactions, raising hands, and live discussions.

Bodily interactions play an important role in the students’ learning (Alibali & Nathan, 2012; Mayer, 2014; Thelen et al., 2001). In on-campus teaching, this is easily accessible. The teacher can use facial expressions, gesturing, and body language to interact with the content on the blackboard, while eye contact between the teacher and students provides the possibility for the students to follow the instructor’s gaze (Fiorella et al., 2019). This helps the students to be more attentive to the presented content.

Previous studies have suggested that students’ attention span during a lecture only lasts for a limited amount of time before it starts to decline (Stuart & Rutherford, 1978), leading to reduced or slower learning. To avoid this, the teacher can break up the lectures by activating and engaging the students by e.g., asking questions or providing tasks or subjects for discussion. The importance of student active learning methods for increasing student engagement has been the subject of many previous studies (e.g., Bonwell & Eison, 1991; Orlov et al., 2021; Prince, 2004).

There are several ways to engage students during the lecture. One simple way is to ask yes/no questions, or other types of questions that the students can respond to by raising their hands. This can both engage the students, keep up their concentration, and at the same time give the teacher real-time feedback that can help to adjust the tempo and teaching methods.

Providing the students with the opportunity to have short discussions (e.g., by providing a math problem) with fellow students is another way to engage students during the classes. Such group discussions help to motivate and improve the social bond among students. The students can think or calculate the problem by themselves before engaging in the discussion. They can also present their work to each other. The teacher can walk around in the classroom to get an overview of how the discussion goes and take questions from students. Often, the students more willingly ask questions when the teacher is nearby.

Research has been done, and the best practices have been found to bring the online environment closer to the classroom environment. For example, a study carried out by Carmichael et al. (2018), shows elements that make education videos successful for students to gain the intended learning outcomes across disciplines. They found that shorter videos or segments on a longer video are beneficial, and the videos that include the teacher’s image are more engaging. They say that videos affect student motivations, confidence, and attitudes positively. The flipped classroom is given as an example of the usage of the videos.

Regarding mathematical education, the flipped classroom is often used as a hybrid version, where the videos are the traditional homework, and the computational exercises are done in the classroom environment with a teacher (López Belmonte et al., 2019; Lopes & Soares, 2018). The factors mentioned in this section have been addressed and considered in the online method described later in this paper.

**Benefits of Classroom Environment When Giving an Exercise Class**

Since pen-and-paper is still most often the fastest and easiest method to write mathematics, the online courses have struggled to give interactive guidance, which has made fully online mathematics courses less successful in comparison with traditional classroom mathematics courses (Trenholm et al., 2019).

Taft et al. (2019) studied the effect of the size of the online class and found it to matter. The large classes are appropriate for foundation-level learning, but small classes are needed for learning requiring higher order thinking. In small class sizes, the teacher-student and student-student interaction, asking for help, and usage of tutoring is easier to accomplish. To empower interaction with fellow students, the students are encouraged to sit in small groups during the exercise classes, so that they can discuss, cooperate, and help each other. They can share their notes to show each other how they solved a problem.

Students will usually raise their hands to signalize the tutor to ask for help. In cases where there are too many students who need help at the same time, a waiting queue can be created by writing their names on the whiteboard. Tutoring on-campus depends on what the students need help with. When they need help to go to the next step in problem solving, they can show their notes to the tutor. The tutor can then help by either pointing out tips in the student’s notes or explaining orally or writing notes that the student can also see.

Most of the distance learning research and examples are about how to make lectures, whereas in mathematics, the importance should be how to deal with exercise sessions online. In the method presented in this paper, the focus is on interaction, queuing system, and tutoring in online classes with small groups. These are discussed in more detail in the following.
Small Tests During the Course and Cheating

In mapping the MOOC education study (Albelbisi et al., 2018), it was mentioned that assessments during the MOOC courses are an important success factor and that weekly quizzes had a positive relationship with the final exam results. However, the types of assessments are to be linked to higher learning. This was found in the study (Fish, 2017), which showed the importance of thinking through what is asked on multiple-choice online tests. Fish found that performing well in multiple-choice questions that include the two lowest levels of Bloom's taxonomy† (Bloom, 1956): knowledge and understanding, did not predict well performance in the later exam.

In addition, online tests, especially when completed at home, open new ways to cheat that cannot be done in the classroom environment, something which was also noticed during the COVID-19 period (Asgari et al., 2021; Bilen & Matros, 2021).

Methodology

In this section, the changes made to keep the benefits of on-campus teaching in online teaching are presented. The pre-calculus course has been used as a test platform. The pre-calculus course is, as the name states, a pre-course before starting engineering-related university studies. This course is meant for those students that are lacking the most advanced level mathematics courses (called R1 and R2) given in Norwegian high schools and are therefore applying via other criteria. Many of them have occupational experience but have the need to refresh the most fundamental mathematics in addition to R1 and R2. The course was newly reformed in 2019, being extended from 6 weeks (with only R1 as curriculum) to 10 weeks. The course has been an on-campus course during all the previous years, and several measures and adjustments have been made throughout that have worked well for the students. This has been evident both during the course and in how the students have managed the later math courses in the engineering programs. The main goal in designing the online course in 2020 was to maintain the successful measures and adjustments that were made when the course was held on-campus.

To participate in the final exam, there are two requirements: a minimum of 80% attendance in classes and a minimum of 80% attendance on weekly tests. Sick leave and other emergency absences are allowed and are not counted in the attendance requirement.

In 2019, when the course was held on-campus, 64 students started the course, of which 59 completed the exam. In 2020, 90 students started the course, of which 79 completed the exam. The course starts at the beginning of June, and admission offers from universities and colleges in Norway are sent out during June, July and the beginning of August. The few students who started the course but did not attend the final exam were mainly students who accepted offers from other universities or colleges during the course.

Data on final exam scores in both years were collected and analyzed to investigate the effect of the on-campus approach on this online course. Since both years had the same type of final exam on-campus, the main difference was whether the course was taught online or on-campus. This makes a clearer comparison of the effects of the approach. In addition, feedback on the courses (both when it was held on-campus and online) was gathered via anonymous surveys at the end of the courses.

Exams were similar and marked similarly in both years, and the score was given in percentages with 100% as a full score. The same applied to weekly tests as they were also graded in percentage. The statistical analysis was conducted with the SPSS software‡. To verify the statistical significance of the results, t-tests were used. As shown in Figure 7 in the findings section, the final exam percentage scores are normally distributed.

The satisfactory level was measured by using a Likert scale from 1 to 5, where 1 was very dissatisfied and 5 very satisfied. The same questionnaire was used in both years.

Preparation Before the Online Course

The following sections address the issues mentioned in the literature review section and explain the preparations made before the course started to make the course start as positive and hinder-free as possible. We have placed much effort into reaching out to students via social media and are prepared to have a plan B when (not if) plan A is not working.

Social Media

The students who attend this course are all fresh students that have never studied at a university before. Even though the university has its own resource site for students, and information emails were sent out before the course started, there might still be problems for some of the students in acquiring the necessary information. This has been a challenge even when the course has been held on-campus.

† https://bloomtaxonomy.net/
‡ https://www.ibm.com/spss
Due to the uncertainty of the COVID-19 situation during the summer of 2020, it was of particular importance that the students frequently got updated information. Commonly, learning institutions use platforms that require a login account that belongs to that institution. Thus, it will also increase the challenge for students who do not manage to have a student account or have trouble logging in to the right platform before the course. One simple solution to this problem is to create a temporary student group on social media, e.g., Facebook†§, which is easily available to most students. Information about this Facebook group was distributed to the students both in the information email and in another Facebook group for all fresh students at the university. It was voluntary to join the group, so those who wanted to join needed to send a request, and then be granted access to the group by the administrator. Almost all the students chose to join the group.

Course-related information was gathered and posted in this group. Several administration staff was also invited to the group with granted access as administrator. They made sure that the students got information about the existence of the group as well as answered practical questions about e.g., applications and registrations. To avoid unnecessary complications on the first day, there was also posted info with pictures on how students could test settings with audio and video in necessary software such as Zoom** and Teams†† before the course started, and even info on how to download these. A voluntary test live streaming was also arranged in Zoom by a tutor, as an opportunity for the students to test their settings.

As an attempt to increase the social relations among the fresh students, a post was made where students could write a short introduction of themselves by replying to the post. All of the 90 students actively participated. The students were encouraged to “like” each other’s introductions, and all the posted introductions got multiple likes. The purpose of this was to create a positive social interaction culture among the students.

To make sure that every student got proper access to the Teams platform with their university account, updating the course information on this Facebook group was continued during the first week of the course. From the second week, the group was left to the students to use as their social group. Additionally, a live welcome and information session was streamed via Zoom at the beginning of the first day of the course. The purpose was to make sure that the practical parts of the preparations were in place before the academic part of the course started.

Always Prepare a Plan B!

In order to get the most out of online mathematics education, it is important to prepare the students before the course, both for the most essential things and backup solutions in case unexpected problems occur. For regular planning, a short animation video was made for that purpose (Jin, 2020). A big challenge for students in online education is to be able to concentrate and focus on their studies in their own homes (Lopes & Soares, 2018). It is important to have a fixed study place in the house with a good and stable internet connection and as few distractions as possible. To maintain the social relation both between students and between students and teacher, as well as to increase the concentration level among the students, they were asked to turn on their cameras during school time. This also contributes to facilitating active learning.

Unlike most other subjects, doing math exercises requires a lot of handwriting. Due to math notations, there is currently no software that can replace handwriting without consuming too much time. Thus, the use of a digital pen is quite necessary for online math education. There is a variety of digital pen options, students can make a pick based on usability and price. After advising, over 90% of the students stated that they had the possibility to write with a digital pen, which made it much easier to tutor mathematics digitally. The students were reminded to download the necessary software on their digital devices both in the video and via posts in the Facebook group. In addition to video-conference tools like Zoom and Teams, the students were informed about handwriting apps like OneNote†§, Microsoft Whiteboard†§ and Explain Everything‡‡. Using these apps on the smartphone works as a plan B solution for most of students and are affordable for those students who do not have a proper PC or digital pen writing tools.

Zoom was the main streaming tool that was used for lectures and exercise classes. If something unexpected was to happen, Teams video conference was the natural plan B. Having both the crew and the students prepared for plan B made the transitions seamless when they were needed. Several other plans were also made in addition, which were rarely in use, and only the tutor crew was informed and prepared for these.

The students were also asked to make an overview of which main hardware and software they used in the course on the front page of their OneNote, in addition to a plan B hardware and software. This way, it was easier for the tutors to provide the proper help when needed. The students were to update their overview accordingly if they had changed their hardware or acquired new equipment.

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† https://www.facebook.com/
** https://zoom.us/
†† https://www.microsoft.com/en-us/microsoft-teams/
‡‡ https://onelab.com/
§§ https://www.onenote.com/
|| https://explaineverything.com/
In addition, the teachers felt that it was important to keep technical assistance separated from the mathematical topic guidance. Therefore, a channel in the Teams room was established (see Figure 1). In this channel, all the students had access to post questions and offer help. Most of the posts were from students who wanted to share useful tips with others in the class. In total, 22 useful technical tips were posted on this channel – from settings in Teams to adjustments in OneNote.

Figure 1. An Overview of the Teams Room

Tutor Training

The tutors play an important role in the course, and it is therefore crucial that they are extra well included and prepared before meeting the students. Four competent tutors were recruited to this course. Several intensive training sessions were held in the week before the course started where different digital solutions were planned and tested. The tutors were thus able to help students with the subject-related questions as well as being able to provide technical support on hardware and software.

A private channel in Teams was created, in which the tutors could share work-related things. Each day a video meeting was initiated (by using the "Meet"-feature in Teams) in the channel, which worked as a digital office during the day. Here all tutors and the teacher had the option to communicate with each other. The meeting was usually initiated by the earliest arriving tutor.

Lectures

The authors will focus on two of the main benefits of on-campus lectures. One is that the teacher can use facial expressions, gesturing, and body language to interact with the content on the blackboard, while eye contact between the teacher and students makes it more engaging for the students. And the other is the benefit of engaging students' active learning through eye contact and verbal communication between the teacher and the students, as well as students' signalization body gestures such as raising their hands to ask questions. In order to make online lectures similar to lectures given in a classroom environment, a newly developed technical method named TeachUs (Jin & Wessel-Berg, 2019; Jin, 2022) was used in combination with Zoom.

TeachUs is a tool that allows a teacher to interact with the content by using body gestures. It gives similar benefits as on-campus teaching, and it also allows opportunities such as displaying other necessary digital content in the same window as the teacher, which gives the teacher opportunities to interact with the other content by pointing or making other gestures. The eye-contact feeling for students is also maintained in this online approach.

Examples are presented in Figure 2. This invention simulates in a realistic way an on-campus lecture with a blackboard, creating an interactive relationship between the teacher and the content that is being presented.
Streaming of the lectures was done in Zoom. Students joined the lectures via a Zoom meeting link with a password. The choice of this software was mainly due to the breakout room feature. In addition, it has several built-in features that can be used to increase interactivity in class. The teachers shared most of their resources and activities in Teams, while all lectures and exercise classes were held live in Zoom with links also posted in Teams.

Student Active Learning

As mentioned in the Literature Review, student active learning methods help to maintain engagement and concentration among the students during lectures. This applies just as much to online teaching as to on-campus teaching. Especially in online teaching, when the students sit in their own homes it becomes easier for them to lose focus and attention.

If students had questions or comments, they could ask verbally through the microphone, similar to what they would do on-campus. In addition, they could write comments in the chat box. It was noticed that during the lectures, most of the students were more comfortable writing in the chat than using the microphone.

Not all the questions can be easily asked by the methods mentioned, e.g., if a student has a question on something specific the teacher has written on the canvas, it can be difficult to formulate orally or via chat. In such cases, the student can access the real-time whiteboard that the teacher uses for digital handwriting content. This way, the students can simply use their digital pen or mouse pointer to mark or write on the relevant site on the canvas. This simulates the on-campus approach, where the student can point to or use chalk to mark on the blackboard.

An option for non-verbal feedback in Zoom is the poll and yes/no-buttons. When using these options, the teacher will get an instant overview of how many students participated, and their answers. For many students participating in yes/no questions or polls seems less intimidating than taking the initiative to write a question or answer in the chat. This is therefore an easy way to engage and include all the students in class. When the teacher has become comfortable enough with the class, he or she can ask randomly among the students that did not participate (if any), if they have another opinion than yes/no. This helps to keep up the concentration level among the students.

Dividing the students into small groups in breakout rooms is the digital approach to discussions in class on-campus. Groups typically consist of 3-4 members. After the first week, pre-assigned groups to a large degree replaced auto-generated groups. The threshold for student discussions is decreased when they get to know their fellow group members. They can present their work by using the sharing function in Zoom if the notes are digital, or simply hold the paper note in front of the camera if it is hand-written. The teacher can “walk through” in a class by joining the breakout rooms. Several types of group discussions, both regular and game-based, were arranged during the lectures to improve students’ activity and engagement.

It was found to be beneficial to reduce the closing room time to 10 seconds instead of the default setting, which is 60 seconds. Thus, when closing the breakout rooms, the waiting time will not be too long before all the students come back to the main room.

Face to Face

Similar to the classroom face-to-face feeling can be achieved in digital lectures if all the students have their cameras on. By using a separate monitor for displaying the student camera images and for the chat, the teacher was able to view the students through the camera at all times.

It might not be easy to convince the students to turn on the camera during the lecture. Therefore, it is important to establish the habit early in the course and make it feel more comfortable and natural for the students to do so. Both in group discussions in breakout rooms in Zoom and during the exercise classes, the students were required to have the camera on in order to achieve a more on-campus experience. It was not required during the lectures, but most of the students nevertheless chose to have their cameras on as they became comfortable with it during the course. Having the camera on also helps with the students’ concentration, and it gives the teacher the possibility to observe the students’ facial expressions during the lectures, similar to on-campus lectures, which is useful feedback for the teacher.
Other Useful Online Features

In each lecture, there was always at least one tutor present who assisted during the lectures. They were given a co-host role in Zoom. Their function was to maintain order and handle all the non-academic-related matters, thus preventing them from taking attention away from the lecture. They were given the ability to grant participants entering the lecture from the waiting room or to remove participants if necessary. Most importantly, if any students needed technical help during the lectures, the tutors could solve the problems one-on-one with the students without affecting the lecture.

By being present in the lectures, the tutors could also easily be updated on the progress of the curriculum, so that they could be more prepared on which questions they would get in the exercise classes which were held right after the lectures. In addition, it provided an opportunity for the tutors to refresh themselves on the course content and the way in which the different subjects were taught or explained by the teacher. Even though it was not required for all the tutors to be present, most chose to stay throughout the Zoom lectures.

Exercise Classes

Unlike many other subjects, having a suitable digital replacement for mathematics tutoring on-campus is quite challenging. The most common tools students use on-campus for solving math problems are the old-fashioned pen and paper. One of the most common methods in on-campus tutoring is the tutor and the student sitting side by side and writing and looking at the same sheet. This way, the students can easily show the tutor what they have done. So far, there are no existing digital solutions that allow for solving math problems in a convenient way by using a keyboard and mouse. The measures this article will address are how to maintain the requirements and benefits of on-campus tutoring. That is, the ability for students and the tutor to solve math problems using handwriting, the ability for students and the tutor to show each other what they have written, and the ability of verbal dialogue and discussion.

The most appropriate solution for math tutoring in digital exercise classes is therefore, free handwriting with a digital pen. After encouraging the students both prior to and at the beginning of the course, 91% of the students had the possibility to write with a digital pen, according to a Mentimeter survey, and it made tutoring a lot easier.

For communication, Teams and Zoom were used simultaneously. For writing mathematics with a digital pen OneNote, Microsoft Whiteboard, and Explain Everything were used.

Interaction With Fellow Students

In the tutoring sessions, the students worked in groups in breakout rooms in Zoom, so that they could use screen sharing or other methods. The groups were pre-assigned, with 4 students in each group. They had both their camera and microphone on while they worked in the breakout rooms.

According to feedback from students, most of them worked well with the group they were in. However, this was not the case in all groups. Therefore, the pre-assigned group lists were adjusted accordingly each week. It worked out well, but it did require extra administrative work for the tutors.

Asking For Help

When the students wanted help from the teacher or the tutors, they could raise their hands digitally by replying to a post in Teams and writing which exercise they needed help with (see Figure 3). The tutors marked the reply with a “like” before they called the student in Teams, so that the other tutors knew which students had already got help. The students could mute themselves in Zoom (without leaving the breakout room) while they were getting help from tutors.

††† https://www.mentimeter.com/
Tutoring

During the video-calls, the tutor had access to the same OneNote page as the student, so that both could write and draw on the same sheet in real-time. As mentioned earlier, the students were to update the information about what software and hardware they were using on the front page of their OneNote, so that it was easier for the tutors to prepare the most suitable technical method. In most cases, the OneNote file was on an iPad/tablet, while the video calls were via PC or smartphone. This method is thus very similar to on-campus tutoring, where students and tutors can sit side by side and write on the same paper.

If delays occurred in OneNote, other real-time digital writing apps like Microsoft Whiteboard or Explain Everything were used as back-up options. Another alternative was simply to let the student or tutor (depending on who was presenting) share their screen in the Teams video call. Then the one not presenting could “request control” in order to “point” to something on the shared screen.

For students who did not have access to a tablet with a digital pen, other options like e.g., a document camera were an alternative. In order to make it more approachable and affordable, the students were given tips on how to set up a do-it-yourself document camera for free. This idea was also presented in a video after the course for future students (Jin, 2021). One last option was that the student took a picture of their work on paper via smartphone and sent it in the chat in the video call.

All the digital tutoring methods might take longer to learn for first-time users, but the teachers experienced that most of the students quickly got a grasp of it. Some students might be skeptical to try online tutoring for the first time, so certain encouragement in the beginning of the course might be needed. Students will get more comfortable with online tutoring after the first time, and it will lower the threshold when they see how actively fellow students are using it.

At the end of the first week of the course, an anonymous survey was given to the students in Mentimeter, in which 41 out of 86 students stated that they had made use of online tutoring. This number clearly increased (to 49 out of 83 according to a new Mentimeter survey) after the second week.

To make sure that the online tutoring was performed in the most efficient way, students were advised to mark their exercises by color, e.g., green if the students managed to solve the problem by themselves with a good understanding, yellow if they did not fully understand the problem, and red if they did not manage the problem. The students could also clarify what they needed help with by commenting or marking the problem on their OneNote page. This way, it was easier for them to see which exercises they managed well and which they had problems with. It also gave the tutor an indication on which topics the students needed most help with, and which topics they needed to work extra on before the tests or the final exam.

Furthermore, the tutor could provide hints or help for the students outside the exercise classes by commenting and marking on the students’ OneNote page. Even though the exercise classes became more hybrid when the COVID restrictions were eased, the online tutoring options remained the same.

Weekly Tests

Weekly one-hour tests have always been a part of the course. It motivates students to study more evenly during the course. The tests are given at the end of each week. A midterm test was held in the 6th week, containing questions from the curriculum spanning all the first 6 weeks. The only permitted aid on the weekly tests (as well as on the final exam) is a two-page formula sheet supplied by the teacher. The test results give students overview/feedback on how
they did during the week. The feedback was handed out at the beginning of the following week with both points and comments.

Understanding is an important part of learning math. Understanding the concept and acquired knowledge is far more important than the final answer. Therefore, it is important that the students show their understanding by step-by-step solutions rather than focusing on the last answer without knowing why. This is important when they are doing exercises and in the weekly tests.

Each test mainly contained questions from the curriculum that had been taught in that week and the questions were thus of various types spanning over the levels of Bloom’s taxonomy (i.e., remember, understanding, apply, analyze, evaluate, and create) (Bloom, 1956), with the main focus on the higher levels. Tasks 1 and 2 below are examples from the weekly tests.

| TASK 1: Given the two coordinate points A(1, 3) and B(5, -1), find the equation for the circle that has AB as diameter. |
| TASK 2: Find the volume of the solid obtained when the place region R is rotated about the x-axis. R is the finite region bounded by $0 \leq y \leq e^x$ and $0 \leq x \leq 1$. |

To solve Task 1, the students first need to find the coordinates for the center of the circle by calculating the middle point of the line AB and the half length of the line AB. Knowledge about vectors was intended to be the method to do this, but the students could obtain additional points if they could find other creative methods. Finally, the equation for the circle could be found. In order to get a full score on this question, the students needed to both illustrate the solutions by drawings and with step-by-step calculations and explanations in order to show their understanding.

Task 2 is based on understanding the principle of finding volume/area by using integration. Since the volume formula was not given on the formula sheet, the students needed to understand how the method of cross sections works in order to set up and calculate the volume by integration.

**Implementation of Online Tests**

When the course has been held on-campus, the weekly tests as well as the final exam have been performed with pen and paper. Proctors are present to assist if students need more writing paper or to remind the students about how much time is left. In addition, they are there to prevent cheating during the test/exam. There is also at least one teacher available in case there are questions or problems regarding the test or exam content.

To have a proper digital replacement of a campus-based test with pen and paper is challenging, especially for a subject like mathematics where understanding is way more important than the final answer. The decided solution was to use Forms together with OneNote.

During the test, all the students had their cameras on in Zoom, as during the regular lectures. Students wrote their final answers in Forms, while all the steps of solving the problems were written in OneNote. Both Forms and OneNote were connected via Teams (see Figures 4 and 5). When the tests were marked, the final points for each question were given in Forms, while all the comments and marking were done in OneNote. It worked out well when over ninety percent of the students had digital pens. Those who did not have a digital pen uploaded pictures of their pen-and-paper solutions to OneNote.

![Figure 4. Weekly Tests in Forms Were Attached as Assignment in Teams, Published in The Main Channel With Notification to All the Students](https://forms.office.com/)
Marking in *Forms* is automatic but allows for manual editing, and it also allows programming of the marking so that each distinct error can automatically deduct the same points. The quiz in *Forms* provided a clear and efficient way of marking, while the final scores that the students got were mainly based on the step-by-step solutions they submitted. There were possibilities to leave comments from the teacher to the students on each exercise in *Forms*, but one inconvenience was that comments and pinpointing had to be placed on the side rather than the place where it was needed. This was especially the case when the solutions were long or contained many steps. However, by combining auto-marking in *Forms* and the possibility to pinpoint on the step-by-step solutions in *OneNote*, the advantage of traditional marking was maintained. At the same time, it increased the marking efficiency.

An info post was posted in *Teams* a short time before each test containing information about which teacher and tutor would be available during the test. This way, the students could send chat messages to the teacher or tutor if they had any questions during the test. This could be either a question about technical support or understanding of the test questions. This is then equivalent to the role of the proctor/assistant in on-campus tests. If many students asked the same questions, they were defined as frequently asked questions. Answers to these were posted as replies under the info post so that other students who had the same problem or question could get the answer they needed.

After each test, an overview was made of the most common errors that the students made. These errors were then reviewed in the first lecture on Monday of the following week.

Furthermore, after each test, the tutors and the teacher made an overview of students who might need extra help or follow-up. In this overview, a list of which topics that the students might need more help with was made. In addition, possible reasons why the students struggled with certain topics were investigated. All the students were offered a one-on-one dialogue session with the teacher in case they needed help with a study technique, especially the students who scored low on the tests. After the test in week 4, an overview of how the students managed the exercises in *OneNote* was made. It was found that, of the 16 students who scored low on the test, 13 did not show exercises in *OneNote*, or only did exercises in the first 2-3 weeks.

**Measures to Prevent Cheating**

As mentioned previously, the purpose of the tests was to give both the students and the teacher a general overview of the students' learning. If the test answers did not represent the students' actual knowledge and understanding (e.g., if they have cheated), the test would lose its accuracy and meaning. It is hard to make all the questions in such a way that they cannot be solved by using existing mathematics tasks solving apps. Therefore, certain measures had to be taken to prevent cheating on the tests.

The most common cheating method in the pre-calculus course is probably the use of apps that can show the step-by-step solution to a given mathematical problem. There are many such apps available. These apps will usually provide the “most elegant/efficient” way to solve the problem. This way, it is possible to get an indication if a student has used such an app. It is important to have an overview of which apps the students know about. To provide this, a survey was conducted in *Mentimeter* (see Figure 6).

Another way to cheat is to communicate/collaborate with fellow students. In order to have a similar test experience as an on-campus test, the students were required to have their cameras on during the tests. As the students were already
used to having the camera on during the lectures and exercise classes, they could perform similarly as if it was an on-campus test.

The students were informed that random samples would be made after each test. If one or several students had performed the exact same solution steps as suggested in the known apps (according to Figure 6), it could lead to suspicion of cheating. The same applied if several students had done the exact same steps, and especially if they had made the exact same errors.

The test scores did not count as a part of the final grade. The main purpose of the tests was to give the students and the teacher general feedback on how well the students had learned during the week. The only requirement for the tests was 80% attendance. Since the tests did not affect the final grade, there would not be anything to gain for the students by cheating. This might be an important factor in the low amount of cheating.

Findings

In this section, the exam results, the relationships between weekly tests and the final exam results and reported student satisfactions are presented and compared. The exam results in 2019 are from the students that took the course in the classroom, and the results in 2020 are from the students that participated in the previously explained online version of the same course.

There were no changes in the curriculum and progress plan from 2019 to 2020. The students had access to the same learning material, and the exam difficulty level was aimed to be similar. The exam was held on-campus with pen and paper both years, with the same permitted aid (a two-page printed formula sheet). Furthermore, for both years, the course had the same main coordinator who taught about 80% of the lectures.

Exam Results

The distributions of the exam percentage scores for the on-campus course (2019) and the online course (2020) are shown in Figure 7. Table 1 compares exam results between the two years. Here the statistical difference is close in favor of the online course. Exam results and satisfactory results indicated that the teaching the students have received in the online course has not had poorer quality than teaching given on-campus. The inventions made in lecturing, exercise classes and tutoring have given positive results. This might indicate that an on-campus approach to an online course does have a positive effect on exam performance.

Figure 6. Word Cloud Showing Which Math Solution Apps That the Students (N = 84) Knew About

Figure 7. Distribution of Exam Scores in Percentage for the Online Class (2020) and the On-Campus Class (2019).
There are of course several factors that can have affected the results. In 2019, the course was newly reformed so that some time for adjustment was needed for the instructors, which might have had a negative effect on the exam performance that year. Letting the students share the digital notes with the teacher and tutors in OneNote in 2020 might also be a motivational factor for students to do more exercises during the course. This could have affected the exam results from 2020 in a positive way.

| Table 1. Mean Values (M) and Standard Deviations (SD) Of the Examination Score in Percentage for the Online Class (2020) and the On-Campus Class (2019) |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|
|                                 | Online          | On-campus       | t-test          |
| Exam score                      | M   | SD | N   | M   | SD | N   | p   |
| 63.31                           | 21.37 | 79  | 54.24 | 20.26 | 59  | 0.013 |

Regression of Weekly Test Results and Exam Results

As shown in Table 2, there is a significant correlation (sig. < 0.001) between the average weekly online test scores and the on-campus final exam score. This means that those who did well in the weekly tests also did well in the final exam. In addition, the correlation is similar to that between the weekly on-campus test scores and exam scores from previous years. This indicates that the measures that were applied in the online tests worked well in order to keep the amount of cheating low, i.e., no higher than on the regular on-campus tests. The relation between the weekly test scores and final exam scores will be further investigated in a future work where approximately one thousand samples of weekly test scores and exam scores from both the pre-calculus and the ordinary calculus course will be analyzed.

| Table 2. Correlations Between Weekly Test Scores and Exam Scores in 2019 and 2020 (b is the Rate of Change in the Regression Equation y = a + bx) |
|-------------------------------|-----------------|-----------------|-----------------|
|                               | N   | r   | SD | b   | sig  |
| 2019                          | 59  | 0.850 | 10.70 | 0.993 | <0.001 |
| 2020                          | 79  | 0.793 | 13.09 | 0.965 | <0.001 |

Survey

After completing the course, the students were asked how satisfied they were with the teaching they had received on a scale from 1 (very dissatisfied) to 5 (very satisfied). As Table 3 shows, there is no statistically significant difference between students that received online teaching in 2020 and the students that received on-campus teaching in 2019. The students were overall very satisfied with the teaching they got.

| Table 3. Mean Values (M) and Standard Deviations (SD) Of the Grade of Satisfaction for the Online Class (2020) and the On-Campus Class (2019) in The Pre-Calculus Course, on a Scale From 1 (Very Dissatisfied) to 5 (Very Satisfied) |
|---------------------------------|-----------------|-----------------|-----------------|
|                                 | Online          | On-campus       | t-test          |
| Satisfaction                   | M   | SD | N   | M   | SD | N   | p   |
| 4.50                           | 0.68 | 58  | 4.58 | 0.61 | 51  | 0.47 |

Discussion

This paper presents a new online course that was created to be more like an on-campus course focusing on interactive teacher-tutor-students communication and pedagogical setup that is suited for mathematics education. When designing the course and its implementation, the authors specifically addressed known lacking and problematic points in online courses as well as applied factors that have been found to be successful in online courses based on the previous studies discussed in the Literature Review section.

The authors addressed the pedagogical and social points of view by adding the possibility of having more face-to-face interaction and communication in online courses, especially in exercise classes and lectures. Being “seen” also online was one factor to keep the students satisfied with the course and keep their motivation to finish the course. Small groups were enabled with break-out rooms for exercise sessions, which made it easier for the students to get to know each other, form social bonds, and feel safe asking for help.

The students could communicate by raising a digital hand to ask a question orally or writing it in the chat. Also, other emojis were used to indicate if the students understood the topic or not, especially when the teacher asked “yes/no” questions from the students. In exercise sessions, the queuing system was used, which proved to be practical and fair, as students got help in the order they asked for it.

It was noticed that the preparation paid off. There were no complaints from the students about unknown schedules, how the course was to be completed, or the lack of knowledge of staff members. The students were also to get hold of
the teaching staff quickly, in addition to getting technical help. Having their own channels for mathematical questions and technical questions helped to keep questions and answers in order, which was beneficial when one needed to look back at the already asked and answered questions to find quick guidance.

The weekly tests used on the online course were designed to demand a higher-level thinking from the students, as encouraged by Fish (Fish, 2017). As the results showed, this was beneficial for the students to achieve higher learning outcomes. The transparency in communication seemed to prevent cheating in tests. In addition to the demand to leave the cameras on, the students were informed that their test answers would be compared to the most common mathematical online tools’ answers (the typing way the tools show the results). As the tests had time limits, rewriting the answers given by the tools would have taken too much time, leaving the students basically two alternatives: solve the tasks on their own, or use the direct copy-paste method, where the latter would then be most likely noticed.

The results found in this paper are positive towards the usage of this type of online course. A similar positive attitude was found in the study of virtual learning environments in mathematics by Bognár et al. (Bognár et al., 2018). In addition, a comparison of the exam results showed that the students studying mathematics online could achieve as high, if not higher, exam outcome as if they were studying the same curriculum in on-campus courses. These results are therefore more optimistic than found in (Trenholm et al., 2019).

Conclusions

In this study, the authors have explained how to build an online course in mathematics that also is able to combine the benefits of traditional classroom education with online education. The results strengthen the evidence found in other studies that in order to deliver successful online mathematics courses both from the teacher and the student perspective, the key factor is to focus on the pedagogic. Technology is made to serve the pedagogic, not the other way around.

As the comparison of the same course delivered both as a classroom and online education shows, the dropout level, satisfaction level, and exam results were at the same level in both educational models. This is evidence that online education can give as good results as classroom education when it is based upon good pedagogical quality, and enough contact platforms and interactions between teacher-student, tutor-student and student-student are built on the educational model.

Recommendations

The following recommendations are made based on the findings of the study and the gained experiences:

- Make a course plan as soon as possible and make it available well in advance before the course starts on platforms that are accessible without having access to the educational institution’s intranet and/or closed systems.
- Even though it is the students’ own responsibility to ensure that they have got the necessary equipment for the online course, there is a large variety in technical experience, background, and economic affordance. Therefore, it is useful to allow different alternatives for hardware and software. In addition to publishing the course plan, it is important to publish information on which tools are applicable and where to get them. For a smooth course start, the students should have technical tools ready before the beginning of the course.
- At the beginning of the course, especially the exercise classes are going to be more time-consuming. Therefore, the teacher and the tutors need to take more initiative to create student-active learning environments at an early stage.
- Create and distribute key information sources (e.g., videos) before the course starts and keep them short.
- Hold question-answer sessions from each topic and show additional examples of how to solve tasks in lectures by handwriting.
- Encourage the students to keep their cameras on during the sessions.
- Use break-out rooms for students to work in small groups.
- Encourage students and provide them with an arena for sharing and exchanging tips and experiences on technical use.

The precautions and preparations addressed in this work can be used as a reference list in preparing online courses. Most of these agree with the points mentioned in (Hadi Mogavi et al., 2021).

Even though we found evidence that online education can give as good results as classroom education when it is based upon good pedagogical quality, there is still a need to focus on online pedagogic. How could online classes be designed to be even more individual and still being student-student and student-teacher interactive? We also encourage future research looking into how technology could be formed to adjust pedagogical needs, and not the other way around.
Limitations

This research is limited to one pre-calculus course class, compared with the same course in the year before. More online classes over a longer period are needed for a more extensive study. And as mentioned under Recommendations several precautions must be made during the practical implementation of the course.

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Jin: Concept and design, data acquisition, data analysis and interpretation, drafting manuscript, critical revision of manuscript, final approval. Helkala: Analysis and interpretation, drafting manuscript, critical revision of manuscript, final approval.

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