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Adapting Bruner's 3-Tier Theory to Improve Teacher Trainees' Conceptual Knowledge for Teaching Integers at the Basic School

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Abstract: The focus of this action research was to adapt Bruner's 3-tier theory to enhance conceptual knowledge of teacher trainees on integer operations. It looks into how learners' conceptual knowledge of integer operations changes over time, as well as their attitudes toward using the 3-tier model. Eighty-two (82) teacher trainees, who were in their first year semester one of the 2020/2021 academic year were purposely selected for the study. Data was collected using test and semi-structured interviews. The study found that using Bruner's 3-tier theory contributed to substantial gains in conceptual knowledge on integers operations among learners. It was also found that learners proffered positive compliments about the Concrete-Iconic-Symbolic (C-I-S) construct of lesson presentation and how it built their understanding to apply knowledge on integers operations. Learners also largely proffered positive image about C-I-S construct as it aroused interest and activated unmotivated learners. On these bases, the study concludes that lessons presentations should mirror C-I-S construct in order to alleviate learning difficulties encountered on integer operations. To do this, the study suggests that workshops on lesson presentation using C-I-S construct be organized for both subject tutors, mentors and lead mentors to re-equip their knowledge and to buy-in the idea among others.

Keywords: *3-tier, conceptual knowledge, integer operations, negative integer, teacher trainees.*

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Introduction

The intrinsic nature of mathematics in social activities has earned its core stance in society. What resonates across the curricula of many countries is the twin premise that "all pupils can learn mathematics and that all need to learn mathematics" (Ministry of Education Science and Sports-Ghana [MoESS-G], 2007a, p. ii). It has enjoyed long-standing prestige, serving as an essential requirement for job-seekers (Watt et al., 2017). Suggestively, therefore, the key to job opportunities is one factor that explains why mathematics has become in demand and a must-learn for many.

One of the fundamental processes of mathematical learning is known as abstraction. According to Skemp (1989), abstraction is extracting what is common to a group of objects or circumstances. Thus, abstraction is key in concept formation. For proper abstraction of a concept or skill, Bruner (1966) and Jones and Tiller (2017) opined that the concept or skill must be described and modelled using concrete objects, using appropriate drawings or pictures to represent it, and finally using appropriate symbols to represent the concept or skill. The concept of integers is one of the abstract ideas that form the basis for many mathematical operations in later years. Knowledge of integers is necessary and serves as a pre-requisite skill in learning algebra and related content areas in mathematics. Nevertheless, there is a gap between what integer concepts are taught and learned compared with what Bruner proposed.

In the primary school mathematics curriculum, integers as a topic are central as they represent a transition from tangible to abstract thoughts. It forms the basis of algebra (Lamb & Thanheiser, 2006). Elsewhere, curriculum standards provide that integer concepts be learned at a 3rd-grade level; other standards disapprove of teaching the concept and its associated operations before 7th grade (Cetin, 2019). In Ghana, integer concepts were not taught until Junior High School One (JHS1) (MoESS-G, 2007a, 2007b), at which point the learner would have attained the average age of 13 years.

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However, in the new curriculum introduced in 2019, integer concepts begin in primary five (averagely at age 10) (Ministry of Education-Ghana [MoE-G], 2019).

According to Sahat et al. (2018), the concept of integers is abstract, and operations on integers involve signs, especially the negative sign, that may qualify the number and the required operations. When asked to perform operations on integers, this dual role of the negative sign frequently causes learners to become confused and frustrated (Sahat et al., 2018). There is a need for rigorous and empirical research into the role of sound methods and theories in how children learn mathematics to address the issue of the duality of negative signs.

In literature, one such practical research technique is action research. The peculiar and unique nature of action research resonates because the researcher is part and parcel of the study. Therefore, the study's significance is contingent on the researcher's correct or incorrect interpretations of the results, personal biases, presentation and analysis style, among others. Due to the centrality of the researcher(s) in action research, a snapshot description of the researcher(s) is necessary.

For more than ten years, the researchers in this study have been involved in teaching practice supervision and imparting knowledge at the university and college levels. However, the nature of mathematics teaching in primary schools worried the researchers. In our observations, teaching for conceptual understanding has fallen by the wayside. Due to a lack of conceptual understanding, many mathematics topics were being glossed over, including integers. Thus, we conducted several action research projects with pre-service teachers, seeking to prescribe remedies to peculiar learning challenges we witnessed with pre-service teacher. Similar learning difficulties with integer conceptualization were observed by the researchers in this context among teacher trainees. Though we had experimented with several strategies with teacher trainees, including the scaffolding model by Bruner, Richard Skemp's primary-secondary concept formation model and Piaget's multiple embodiment theory, this paper reports on only Bruner's 3-tier model.

Problem Statement

Strong background knowledge of integers is one of the requirements for success in algebra and other related areas of mathematics (Cetin, 2019; Kubar, 2012). However, conceptualizing and teaching integers, particularly in comparing and performing operations, are challenging for teacher trainees (Cetin, 2019; Kubar, 2012). The challenges of these teacher trainees in teaching integers emanate from several sources, including the dual meaning of the "—" symbol (Bofferding, 2014). For instance, literature provided by Bofferding (2014) acknowledged that the minus sign "-" could carry binary or unary meaning depending on its usage.

Another challenge is ambiguous integer expressions that refer to "quantity" and "direction". The problem is that when "+" or "-" are used to indicate quantity and direction, the line between the two becomes hazy (Spang, 2009).

The Integers concept marks a transition from concrete to abstract (Cetin, 2019). Therefore, in attempting to model the concept, some teacher trainees use inappropriate concrete models, while others completely abandon the use of models. Instead of serving to clarify, some models impede understanding, because of that some teachers go solo instead of using models. Teachers frequently instruct students using a rule-based, show-and-tell method that involves making them memorize concepts. This action leads learners into learning procedures rather than completely understanding the topic.

In order to avoid telling learners to memorize procedures and rules, pre-service teachers need to use suitable teaching strategies tailored to learners' level of understanding. One strategy is to model integer concepts in consonance with Concrete-Iconic-Symbolic (C-I-S) modes. Bruner's (1966) styles of representation capture the usage of learning resources and models. Bruner developed three modes of representation. These include enactive/concrete, iconic, and symbolic representations (Cabahug, 2012). According to Akayuure et al. (2016), representations significantly increase students' conceptual understanding. Yet there is limited use of learning sources and models in concept representations by teacher trainees.

In learning mathematical concepts like integers, Bruner's 3-tier theory of concrete, iconic, and symbolic may be constructive (Cabahug, 2012; Currell, 2021). These research findings formed the basis for this study.

Purpose of the Study and Research Questions

The main purpose of this study was to adapt Bruner's 3-tier theory to enhance teacher trainees' conceptual knowledge on integers operations. In pursuance of this purpose the following research questions were formulated to guide the study;

1. W**hat is the change in learners' conceptual knowledge on integers operations following the use of Bruner's 3-tier theory for teaching and learning**?
2. What views are expressed by learners following the use of models and or manipulatives in learning and teaching of integers?

In answering the first research question the hypothesis below was formulated;

H_0: There is no statistically significant difference in participants' mean scores for the pre-test and post-test.

Literature Review

Theoretical Underpinning

The theoretical foundation for this study is Bruner's modes of representation, which strongly emphasises constructivist learning theory. As a learning theory, constructivism focuses on how people learn new things based on previous experiences and observations of the world (Ampadu & Danso, 2018). The teacher's job is to help students develop knowledge and understand what they have learned. Bruner's 3-tier theory of representation explicates that any time new material is encountered by a child, the child's mind configures the new material in three phases: enactive, iconic, and symbolic (Drummond, 2021). The first tier is the enactive stage. In the enactive stage, the child understands a concept through the support of concrete actions. Learners begin to develop understanding through active manipulation (Drummond, 2021; Jones & Tiller, 2017; Kamina & Iyer, 2009). Therefore, in teaching a concept, the child should be allowed to "play" with the manipulatives that embody the concept to understand how it works fully (Kamina & Iyer, 2009).

The second tier is the iconic stage. At the iconic stage, learners can create mental pictures of the information; thus, physical manipulation is no longer required (Bobek & Tversky, 2016; Drummond, 2021; Kamina & Iyer, 2009). Learners can visualise factual information through pictures, as some pictures are informational, helping to narrow the understanding gap (Bruner, 1966).

In the last and final stage, which is the symbolic stage, the child now only acts upon symbols (Drummond, 2021). Students' ability to depict the world using abstract concepts characterises this stage. Learners at this stage can assess, critique, and reason appreciably.

A stellar point picked from Jerome Bruner's study is that the child's mind grows through 3-tiers of learning: enactive, iconic, and symbolic. Though the emphasis here is on children, it also applies to adults. Bruner believes that to develop their understanding, children (learners) should experience all these stages sequentially. Bruner contends that effective teachers must support and guide students in completing the three significant stages through a process he refers to as scaffolding, in which students build understanding. Scaffolding, in the end, permits students to become self-reliant learners (Aduko, 2016).

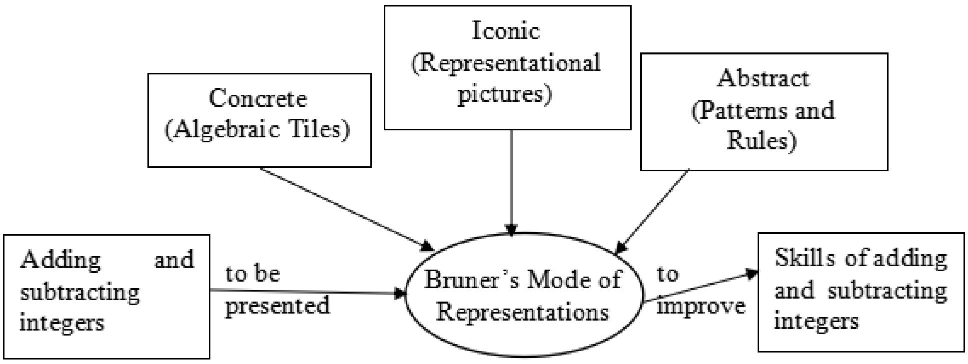

Figure 1. The Theoretical Framework

Instead of rote learning, the goal of classroom learning is for teachers to assist students in developing their understanding (Drummond, 2021). Learners would then be able to understand and filter new information based on their prior knowledge. According to Bruner, reorientation of mental schema occurs when learners can connect their newfound information to what they already know (Bruner, 1966). This mental reorientation gives the new information meaning and allows the learner to think beyond it (Drummond, 2021). To Bruner, "knowing is a process, not a product" (Bruner, 1966, p. 72). As a result, learning will become more critical, practical, and outstanding if teachers present information in line with the Concrete-Iconic-Symbolic (C-I-S) construct and encourage learners to leap into the imaginable. Teachers will be doing an injustice to teaching if they continue to present facts instead of teaching to open up questions (Blazar & Kraft, 2017; Sherwood, 2005).

Conceptual Knowledge

Conceptual understanding refers to a learner's capacity to think in situations that require the cautious use of definitions or relations (Al-Mutawah et al., 2019). Learners can identify, label, and produce examples of concepts; they can use and compare models, illustrations, and manipulatives to show their conceptual understanding of mathematics (Al-Mutawah et al., 2019). Additionally, conceptual mathematical understanding is demonstrated through the use of mixed examples, the recognition and application of principles (MoESS-G, 2007b; National Council of Teachers of Mathematics [NCTM], 2000), the identification, interpretation, and application of signs, symbols, and definitions, and the comparison, contrast, and combination of related ideas and principles (MoESS-G, 2007a, 2007b; NTCM, 2000).

Conceptual understanding is one of the three important strands that define mathematical proficiency; the others are problem-solving and procedural knowledge (NCTM, 2000). To this end, the emphasis of classroom instruction is on conceptual understanding as a necessary means. Most importantly, learners must study mathematics with comprehension, enthusiastically constructing new information from existing facts and experience (Cabahug, 2012).

Methodology

Research Design

Research design is the framework for a researcher's methodologies and strategies to conduct research (QuestionPro, 2021). It permits researchers to focus on research methodologies appropriate for the study and establishes a successful research path. The study was action research, which follows a pre-test-post-test design, using interviews to provide explanations. As Devault (2020) argued, merely focusing on numbers risks one losing relevant information or themes that could benefit the study immensely.

According to Bryman and Bell (2011), action research is often suitable when teachers desire to take immediate steps to solve a classroom problem. It is typically conducted in a local setting and is context-specific. Moreover, the design allows one to combine qualitative and quantitative data to obtain thorough knowledge concerning a problem (Collis & Hussey, 2003). The problem-solving features of action research make it crucially relevant for investigating learning challenges in the classroom. A significant setback in using action research is its lack of repeatability and rigour (Bryman & Bell, 2011). However, the focus of action research is a solution in orientation and not the reproducibility of results (Hughes, 2008), as problem situations and contexts differ. Working to achieve practical results is thus the general goal of action research.

Sample and Data Collection

The research population consisted of all the 346 first-year teacher trainees at Gbewaa College of Education, Pusiga, for the 2020/2021 academic year. The sample was selected via both convenient and purposive techniques. First, one entire group consisting of one hundred and seventy-two (172), to which one of the researchers was assigned, was conveniently selected. From this number, eighty-two (82) were purposively selected based on their performance in a diagnostic test.

Purposive sampling's primary purpose is to concentrate on specific population characteristics relevant to the research topics at hand (Laerd Dissertation, 2022). Since the object of this study was to assist those learners (teacher trainees) challenged with integers and integer operations, purposive sampling best suited the intent, as probability sampling may exclude those learners who most need the assistance. Data was gathered using pre-and post-tests and semi-structured interviews.

Pre-Intervention Procedure

A pre-test was conducted to obtain baseline data on learners' ability to do basic integer operations. Test items were picked based on the course syllabus (Ministry of Education-Ghana, n.d.) to ensure content validity. The new course syllabus integrates both content and pedagogic knowledge. In order to ensure construct validity, all the items, nine (9) in number, were designed to test conceptual knowledge of integers but on different domains. Items 1, 4, and 7 were based on knowledge and understanding (KU) of integers. Items 2, 5, and 8 tested application of knowledge (AK), while items 3, 6, and 9 tested pedagogic knowledge (PK). Earning criterion validity means ensuring that the items reflect the learners' KU, AP, and PK. A maximum of five minutes is allocated to each question, making an overall duration of 45 minutes for the entire test. The post-test items were set to follow a similar trend, but they were not the same items. The purpose of not using the same test items was to ensure validity was not sacrificed on the altar of reliability.

After the pre-test results were scored, five teacher trainees were purposely selected based on their performance and interviewed following a semi-structured interview format.

Intervention Procedure

The intervention lasted five weeks and followed the 3-tier learning theory of Jerome Bruner, namely the enactive stage, the iconic stage, and the symbolic stage. In teaching, the teacher starts with concrete actions or concrete materials. Next was using pictures or drawings and, finally, abstract operations. For each week, the researcher met the learners two times. Each meeting session lasted 60 minutes. The meeting days were Tuesdays and Fridays from 4:00-8:00 pm. The intervention lasted five weeks and followed the 3-tier learning theory of Jerome Bruner, namely the enactive stage, the iconic stage, and the symbolic stage.

Week 1: Integer Chips

Integer chips were used as concrete scaffolds to help learners grasp the concept of integer addition and subtraction. As shown in figure 2a, yellow chips represented positive integers, and red chips denoted negative integers.

Figure 2a. Positive and Negative Integers Models

For instance, the modelling for 4, -1, and -3 are illustrated in figure 2b.

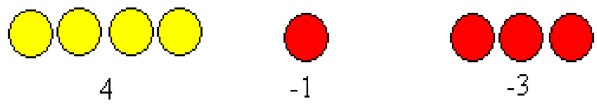

Figure 2b. Exemplars for Positive and Negative Integers Models

Knowing how to model a zero, or the centre of an integer, is crucial. We have a zero-pairing situation when we have the same number of yellow and red chips. For example, the image below depicts zero pairs.

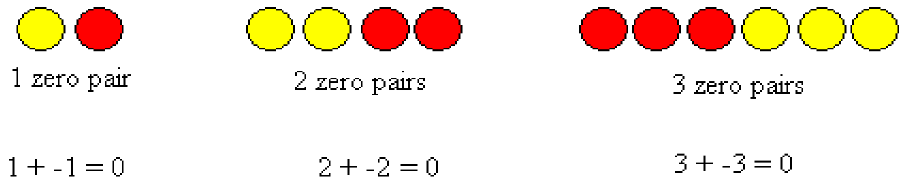

Figure 3. Modelling Zero Pair with Integer Chips

Adding and subtracting integers with modelling is helpful for learners with difficulty understanding integers. Adding and subtracting come as necessary physical operations when modelling integers, in line with Bruner's phase one of the 3-tier models. If the operation/action using the chips is carried on a display board, adding implies "add something to the board," while subtraction refers to "remove something from the board". At this point, we started by using a big square to represent a board.

Example 1: (-4) + (-3)

The researcher demonstrated this by putting four red chips on the board. They were followed by three red chips on the board again because all the numbers in the example are negatives. The correct result is -7 since we only have 7 red chips.

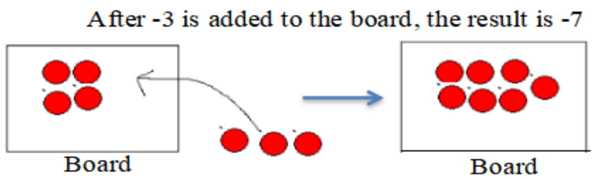

Figure 4. Modelling (-4) + (-3)

Notice that the big arrow represents the "+" sign or the action of adding.

Example 2: (-3) + 2

Add 3 red chips on the board to represent -3. Then, add 2 yellow chips to represent 2.

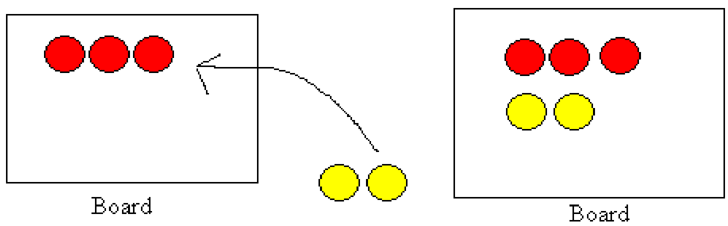

Figure 5a. Modelling (-3) + 2

Get rid of the two zero pairings on the board. The result is -1 because there is just one red chip left.

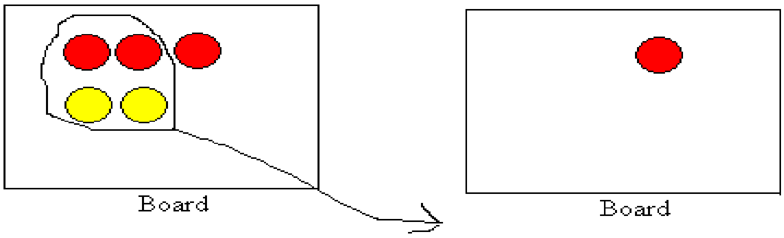

Figure 5b. Final Model for (-3) + 2

Example 3: (-4) – (- 2)

This example was demonstrated by asking learners to symbolize -4 and place four red chips on the board. The problem interpretation then tells you to take two negatives away. Take away two red chips (-2) from the board by subtracting (minus) negative two. Consequently, the result is two red chips, interpreted as (-2). Thus (-4) - (-2) = (-2).

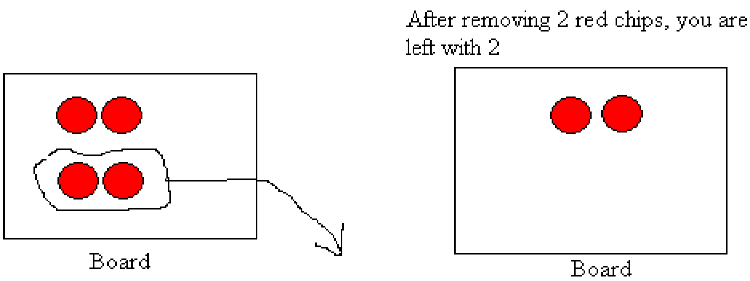

Figure 6. Modelling (-4) - 2

After these concrete representations and actions, learners solve similar problems. Example (+3) + (-7) by drawing pictures of 3 yellow chips together with 7 red chips. They form 3 zero-pairs, and the remainder is 4 red chips.

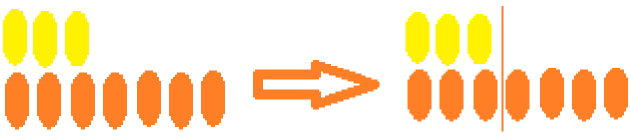

Figure 7. Model demonstrating 3 + (-7)

Thus, from figure 7, it can be seen that (+3) + (-7) = (-4)

After sufficient practice with this, learners (teacher trainees) were encouraged to solve similar problems, this time round, by forming mental pictures of the chips involved without drawing. For example, to find (-3) - (+5), learners imagine 3 red chips combined with 5 yellow chips. After forming 3 zero pairs, the remainder is 2 yellow chips or (+2). Hence (-3) - (+5) = 2.

Week 2: Charged Particle Model

A model of charged particles is followed by an explanation of how the model works. Charged particles were used to facilitate the addition and subtraction of integers. A card with the plus sign (+) embedded represents positive integers, and a card with a minus sign (-) embedded represents negative integers, as shown in figure 8.

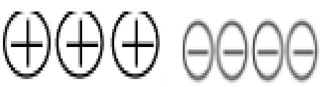

Figure 8. Charged Particle Model of Integers

Examples were then provided to illustrate how the model works. It is similar to the chips, with the additional feature that learners are made to picture an atom, either positively charged (+) or negatively charged (-).

Example 1: To solve 3–(–4), learners begin with 3 positive particles. Because no negatives appear at the start, one must eliminate 4 of them (i.e., 4), and put in four pairs of positive and negative particles, corresponding to four zeros.

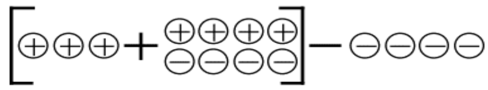

Figure 9a. Charged Particle Model for 3 -(-4)

Remove 4 negatives from the zeros just added, leaving you with the original three positives and the 4 positives from the zeros left, which are then added to gives 7 as the final result.

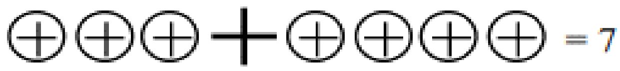

Figure 9b. Charged Particle Model for 3 + 4

This really shows why 3 – (–4) = 3 + 4.

After using concrete models, the learner moved to 2nd stage of Bruner's 3-tier model, the iconic stage, where they drew pictures of the charged particles. For instance, to solve (+5) + (-4), learners draw 5 positive-charged particles together with 4 negative-charged particles. Next, they formed 4 neutral charged particles (4 zero pairs), leaving (+1) as the net charge. Therefore, (+5) + (-4) = (+1).

After sufficient practice at this stage, learners navigate to phase 3 of Bruner's 3-tier theory, where they form mental or abstract pictures of action and solve the particular problem. For instance, to find the difference between (-5) and (+4), i.e., (-5) - (+4), learners imagine (without drawing) 5 negatively charged particles together with 4 positively charged particles. They further picture 4 positive and 4 negative particles neutralising to form a neutral charge (4 zero pairs) remaining one negative charged particle as net charge and conclude that (-5) - (+4) = (-1).

Weeks 3: The Number Line Model

Week 3 was devoted to the use of the number line. For example, using the number line to solve addition of (-2) + (+4) and (+3) + (-5) and also subtraction of (-2) – (+1) and (+2) – (+6). First, the number line was drawn on the concrete floor and labelled from zero to (+12) on the right side and from (-12) to the left side of zero, respectively.

Explanation: Zero (0) is the centre of the number line and is neither positive nor negative. Figure 10 depicts negative numbers to the left of zero and positive numbers to the right.

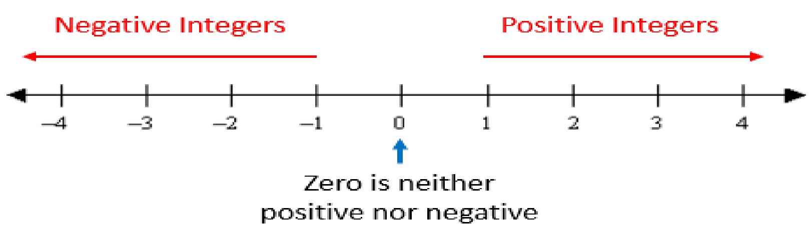

Figure 10. Number line Model

Example 1: Find: (-2) + (+4)

Instruction: From zero on the number line, facing the left side (negative direction), move 2 units forward. Turn right facing the right side (+) and move 4 units forward (+). The final destination is (+2). meaning that (-2) + (+4) = +2.

Example 2: Add (+3) and (-5)

Instruction: From zero on the number line, facing the right side (positive direction), move 3 units forward. Still facing the same direction (+), move 5 units backward (-). The final location is (-2). meaning that (+3) + (-5) = (-2).

Adding or subtracting numbers on a number line is a neat way to see how numbers are added or subtracted using visual interpretations.

Example 3: Find (-2) – (+1)

Instruction: From zero on the number line and facing a negative direction, move 2 units forward. Turn, facing the right side, and move one unit forward. The final location is (-1). Hence, (-2) – (+1) = (-1).

Example 4: Find the difference between (+2) and (-6).

Instruction: From zero on the number line and facing a negative direction, move 6 units backwards. Turn, facing the right side, and move one unit forward. The final location is (+8). Hence (+2) - (-6) = (+8).

Next, the number line was drawn on a marker board and used to solve given problems. Learners were then encouraged to draw pictures of the number line in their exercise books to solve given integer operations. After that, learners formed pictures of the actions without drawing number lines. For instance, to subtract (-2) from (-5), the learner is pictured facing the negative direction on the number line from zero and moving 5 units to -5. He/she turns facing right but moves 2 units backwards to (-7). Thus, (-5) - (-2) = (-7).

Week 4: Multiplication of Integers Using Charged-Particle Model

Example 1: Find 6×2 (multiplying 2 positive integers)

Let pupils to make 6 lots of 2 positive charged particles.

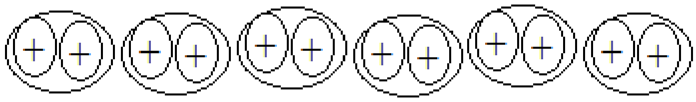

Figure 11. Charged Particle Model for 6×2

In all there are 12 positive charged-particles. So, $6 \times 2 = 12$

Example 2: Find $(-6) \times 2$

This can be interpreted as 2 makes lots of 6 negative charged-particles.

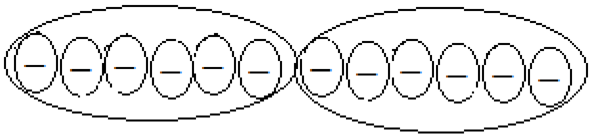

Figure 12. Charged Particle Model for $(-6) \times 2$

Altogether gives 12 negative particles. Thus $-6 \times 2 = -12$

Example 3: Find -6×-2

Guide pupils to interpret -6×-2 as remove 6 groups of 2 negatively charged-particles.

Pupils start by modelling enough neutral particles, from which they then remove the 6 groups of 2 negatively charged particles.

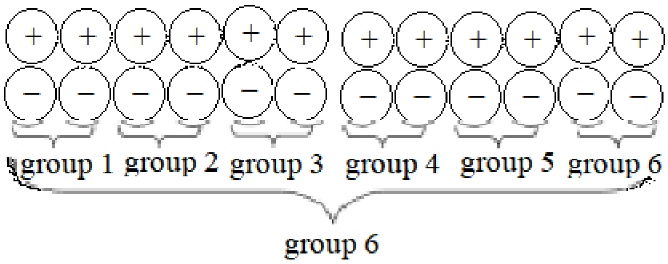

Figure 13a. Charged Particle Model for $(-6) \times (-2)$

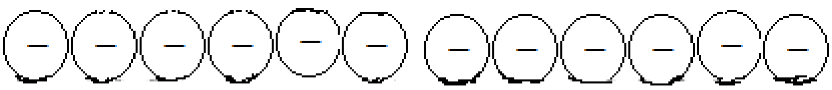

Figure 13b. Final Model for $(-6) \times (-2)$

The final diagram now has 12 isolated positively charged particles as a result of $(-6) \times (-2)$ i.e. $(-6) \times (-2) = 12$

Using The Number Line

Learners act out the multiplication of integers by drawing the number line on the floor. Then they draw on their exercise/notebooks to solve multiplication problems.

Example 1: Using the number line, show and describe briefly how you will help a junior high school pupil find the product (+2) (-3).

Explain to the pupils that; from zero, face right and hop/walk backwards 2 places three times to land at (-6) as seen in the number line. (+2) $\times$ (-3) = (-6) interpreted as 0 – (+2) – (+2) – (+2).

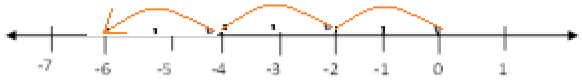

Figure 14: Number Line Model for (+2)$\times$(-3)

Next, learners solve multiplication problems mentally (abstractly). For instance, learners interpret (-2)$\times$3 as from zero faces left, walk/hop forward 2 places three times, that is (-2) + (-2) + (-2) = (-6).

Week 5: Division of Integers Using Charged Particles Model

Find -12$\div$3. This may be interpreted as 3 pupils sharing 12 negative charged particles, how many will each receive?

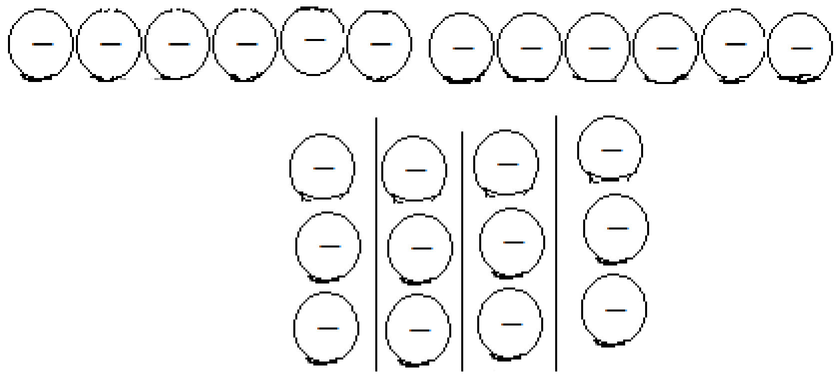

Figure 15. Model for -12$\div$3

We shall use step by step computation (algorithm) to solve division problems of the form 12$\div$ (-3) and (-12) $\div$ (-3).

For (12) $\div$ (-3);

Let $y = (12) \div (-3)$ ....(1) — Equating the expression to y

$-3y = 12$ — Rewrite equation (1) as an equivalent multiplication sentence

$\left(-\frac{1}{3}\right)(-3y) = 12\left(-\frac{1}{3}\right)$ — Multiply both sides by multiplicative inverse of (-3)

$1 \times y = \frac{12}{3} = 4$ — But any number times 1 is same number, except zero

Thus $y = 4 \Leftrightarrow (12) \div (-3) = -4$

For (-12) $\div$ (-3);

Let $y = (-12) \div (-3)$ ....(1) — Writing (-12) $\div$ (-3) as a division equation in y

$-3y = -12$ — Rewrite equation (1) as an equivalent multiplication sentence

$\left(-\frac{1}{3}\right)(-3y) = -12\left(-\frac{1}{3}\right)$ — Multiply both sides by multiplicative inverse of (-3)

$1 \times y = \frac{12}{3} = 4$ — But any number times 1 is same number, except zero

Thus $y = 4 \Leftrightarrow (-12) \div (-3) = 4$

Example: Show how you will help pupils in Junior Secondary School to discover for themselves that 5$\div$0 is undefined.

Help pupils to let $5 \div 0 = y$ and express it as $y \times 0 = 5$

Ask pupils to find the values of y that makes the mathematical sentences true. Explain to pupils that since there are no value of y that makes the sentence true it shows that $5 \div 0$ is undefined.

Post Intervention Procedure

After the intervention, a post-test was conducted, which followed the same pattern in scope, content, and procedure as the pre-test. The essence was to compare the margin of change. Five (5) learners (teacher trainees) were purposively selected and interviewed based on their performance in the two tests.

Analysing of Data

Research question one was analysed descriptively by comparing the results with the criteria outlined in Table 1. The study generated qualitative as well as quantitative data. Descriptive statistics (including means, percentages, and frequencies) and inferential statistics, particularly paired-samples t-test, were employed in analysing the quantitative data.

Table 1. Grade Ranges and Interpretations

Score Range	Grade	Interpretation
0-34	F	Fail
35-44	E	Weak pass
45-59	D	Credit pass
60-69	C	Good
70-79	B	Very good
80-100	A	Excellent

Source (MoESS-G, 2007b, p.xiii)

We ensured the data were independent, measured on an interval or ratio scale, and had a normal distribution before using the paired-samples t-test. The effect size was calculated using Cohen d. Cohen's (1988) rule of thumb for effect size magnitude which states that an effect size of 0.2 is small, 0.5 is medium, and 0.8 or greater is large.

The qualitative data from interviews was audiotaped with the consent of the participants, transliterated and then analysed using the thematic approach. For qualitative data, reliability is parallel to dependability (Lincoln & Guba, 1985; Nowell et al., 2017). Dependability discusses how stable the results of a study are over time and answers the question: will the results of a study be the same when replicated with the same or similar respondents in a similar context? Reflecting upon this question and in the context of this study (action research), our concern was the latter in ensuring dependability. That is, if replicated under similar respondents. One method used to ensure confirmability and lessen the impact of researcher bias and thus increase reliability was data triangulation. Again, an audit trail, which describes the detailed methodological or step-by-step description of how data was gathered and processed in this current study, is given to ensure repeatability.

Findings / Results

The pre-and post-test scores were collected and analysed in order to answer research question one (#1): What is the changes in learners' conceptual knowledge following the adaptation of Bruner's 3-tier theory for teaching and learning? The pre-test and post-test results were compared in line with the assessment criteria of MoESS-G (2007b) in Table 1.

Five of the 82 learners were interviewed in order to explain the learners' performance as well as obtain the views of learners to answer research question two (#2): What views are expressed by learners following the use of models and or manipulatives in learning and teaching of integers?

Research Question #1

One of the research objectives was to find out the change in learners' conceptual knowledge of integer operations following their participation in the intervention involving the application of Bruner's 3-tier theory. In doing this, learners' pre- and post-test results were compared as shown in Table 2.

Table 2: Comparison of Performance in Pre- and Post-Tests

	Pre-Test		Post-Test	
Score Range	No. of Learners	%	No. of Learners	%
0-34	62	75.61	0	0
35-44	20	24.39	0	0
45-59	0	0	4	4.88
60-69	0	0	9	10.98
70-79	0	0	43	52.44
80-100	0	0	26	31.71
Total	82	100	82	100

Table 2 presents the level of performance of learners on integer conceptual knowledge. When the performance is compared to the evaluation criteria in Table 1, it is clear that most students (75.61%) failed, with the remaining students receiving only weak passes (35–44%) on the pre-test. The learners in this study scene had six years exposure to integers concept and operations. They were first exposed to integers from JHS 1 to SHS (Senior High School). Therefore, learners' not too impressive performance in the pre-test indicates that they had ill-constructed conceptual knowledge of integers. Meanwhile, these learners would soon be teaching the same concept to primary school pupils, making it compelling to have themselves purged of this defect.

The post-test results in Table 2 show a considerable improvement as the lowest score range was 45–59%, with only 4.88% of the learners scoring within it. According to Table 1, this scoring range is credit pass, which registered no score in the pre-test. Thirty-one point seven-one per cent (31.71%) of the scored marks were in the highest score range of 80–100%, representing excellent performance. It can therefore be concluded that the learners had substantial gains in conceptual knowledge following the intervention.

Table 3 presents other descriptive statistics that help to compare the two tests and to appreciate the margins of gains.

Table 3: Further Descriptive Statistics from Pre- and Post-Tests

Statistics	Pre-Test	Post-Test	Gain
Mean	27.74	75.56	47.82
Maximum	44	90	64
Minimum	15	55	32

From Table 3, the mean score in the pre-test is 27.74%, and that of the post-test is 75.56%, with a gain mean of 47.82%. The pre-test has maximum and minimum scores of 44% and 15%, respectively, both of which are considered failures. On the other hand, the post-test mean score is 75.56%, with a maximum of 90% and a minimum score of 55%.

In addition to descriptive statistics, the researchers found it helpful to use inferential statistics to help infer some findings about the population from the sample. The paired-samples t-test was used in particular. Table 4 shows the results of the paired-samples t-test on the participants' performances. As shown in Table 3, a comparison of the mean scores from the two tests indicates that participants' post-test mean of 75.56 was better than their pre-test mean of 27.74. This suggests that the participants performed better in the post-test than in the pre-test.

To test whether this difference in mean scores was statistically significant, the researchers ran a paired-samples t-test. Table 4 shows the results of paired-samples t-test and reveals a statistically significant difference in participants' mean scores for the pre-test ($M = 27.74, SD = 8.588$) and post-test ($M = 75.56, SD = 8.389$) conditions; $t (81) = -61.585, p = 0.000 < 0.05$. According to Cohen d's (1988) rules of thumb on effect size magnitudes, an effect size of 0.2 is small, an effect size of 0.5 is medium, and an effect size of 0.8 or more is large. The result in Table 4 shows an effect size of 6.801, indicating that the extent of the difference between the mean scores of participants on the pre-test and the post-test using Bruner's 3-tier theory was large.

From the paired-samples t-test result, the null hypothesis that there is no statistically significant difference in the participants' mean scores for the pre-test and post-test is rejected.

Table 4. Paired-Samples t-test of Participants' Pre-and Post-Tests

	N	Mean	Std. Dev.	Mean Difference	t-value	df	p-value	Effect size
Pre-test	82	27.74	8.588	47.82	-61.585	81	0.000	-6.801
Post-test	82	75.56	8.389					

Research Question #2

In answering research question #2, five learners were interviewed. They are identified here as Learner #1, Learner #2 … Learner #5, thereby concealing their real names. These five learners were among those who obtained low scores in the pre-test, but their performance appreciated substantially in the post-test. The researchers were curious to discover their views on participating in the intervention activities. It was necessary to determine their views before and after the intervention. The excerpt below presents the views of the learners:

Researcher: What do you say about integer operations?

Learner #1: 1 The whole idea is difficult to me. e.g., I don't understand why (-7) +5 $\neq$ 12. I only 'chewed' it.

Researcher: Why is (-7) +5 not (-2)?

Learner #1: 2 If we consider "-", the minus, the answer is 2 and not (-2). So confusing! Because of this, I can't and won't want to teach mathematics.

Learner #2:	3	I didn't have interest in operations on integers. Hmm…see how can you say that (-2) × 3 = -6? Why?
Learner #3:	4	Lessons on integers are boring to me. I'm always hoping that we jump to something else.
Learner #4:	5	I was bored when I was told that 12 ÷ (-3) = -4. The whole explanation right from the beginning was not clear
Learner #5:	6	Confusing and making no sense to me and hard to understand and explain.

The views expressed suggest that learners were clueless about integer operations and treated it just like operations on whole numbers. What are the causes of their concerns surrounding integers? Below is an excerpt of some of the learners' responses:

Learner #1:	7	I don't know when "-" is called "subtraction" and when it is called "negative".
Learner #2:	8	I personally don't like the topic. I don't like mathematics.
Learner #3:	9	I don't understand why 2 - (-5) is 2 + 5. I only kept quiet.
Learner #4:	10	Ordering is difficult for me because I could not see why (-3) is greater than (-7). How?
Learner #5:	11	I don't understand "negative" and "minus"

The responses provided by learner #1 and learner #5 corroborated Bofferding (2014) on the duality of the meaning of "-". When not correctly modelled, there is a greater propensity for misconstruing the unary and dual interpretation of the minus sign. Some of the teacher trainees also came with the mindset that they do not like mathematics, as reflected in learner #2. For these learners, more needs to be done apart from the remedial teaching provided through this action research, considering that these teacher trainees would be teaching mathematics at the primary level, where they are being trained to ply their trade. Mathematical skills are not only core but also a critical filter for job seekers (Watt et al., 2017). Therefore, addressing these unfavourable perceptions of "I dislike mathematics" and "I cannot do mathematics" is crucial before these trainees leave their training institutions and begin their careers as teachers.

Following the learners' participation in the intervention, it was notable to figure something out once again about their views through another interview.

Researcher:		Tell me your opinion about the lessons that you participated in.
Learner #1:	12	Interesting.
Researcher:		Can you explain that? What aspect was interesting?
Learner #1:	13	The whole thing.
Researcher:		Hmmmhu….
Learner #1:	14	I had difficulty understanding and carrying out operations like (-5) + 2 but after modelling the operation using integer chips, I was fine.
Learner #2:	15	The entire lessons were interesting. I'm eager to teach my class same.
Researcher:		You mean when you start teaching?
Learner #2:	16	Yes, and I'll teach integers during micro-teaching.
Researcher:		So the lessons are not boring?
Learner #2:	17	No. This style for teaching was understanding. We do know why.
Researcher:		Can you explain to me (-3) × 4? Use any strategy of your choice.
Learner #2:	18	Okay. I can use integer chips but I'll use number line. So to do this, start from zero on the number line and face negative direction. Hop 3 units each forward 4 times. Your final location is (-12).
Research:		Great!
Learner #3:	19	I enjoyed it.
Researcher:		Hmmhu…so was it boring?
Learner #3:	20	No, not boring at all. At first it was difficult for me to carry out 2-(-5) but following the use of the integer chips and charged particles, I got it well.

Researcher: I see can you explain to me 2- (-5)?

Learner #3: 21 Yeah. We are to subtract/take away 5 red chips (-5) from 2 yellow chips (+2) but it is not possible. So I will add 5 zero pairs to it. Then I will now take away the 5 red chips. This leaves a total of 7 yellow chips as the answer. So 2 - (-5) = 2 + 5 = 7

Researcher: Waaaw! Can you explain this to basic school pupils?

Learner #3: 22 Hahaha…. Yes, I can.

Learner #4: 23 Sir, the whole thing was interesting. I've explained it to my group members and they wanted to come and join.

Researcher: Okay! I see. So explain to me how to find (-15) ÷ 3.

Learner #4: 24 This was difficult for me but it's easy to explain now. So you make (-15) negative charged particles. Then share it among 3 groups. Each group will have 4 negative charged particles. So (-15) ÷ 3 = (-4)

Learner #5: 25 The lessons were exciting. If we were to be learning like this, …. Hummm. I wouldn't have felt this bad toward mathematics.

Researcher: Can you interpret the meaning of the minus sign in 7 - (-2)?

Learner#5: 26 Sir, from left to right, the first minus sign is binary connecting 7 and (-2) meaning 'subtract'. The next minus sign means negative 2, which unary. So we are to subtract negative 2 from 7.

Researcher: Beautiful. Okay. What next?

Learner #5: 27 Since there are no negative from 7 to take away, I'll add 2 zero pairs from which I'll take away 2 red chips (-2). The result will be 9. So 7 - (-2) = 7 + 2 = 9.

Researcher: I see that you are using the chips but you didn't draw them. Why?

Learner #5: 28 Yes, if you understand how it works, you can do it mentally and get the answer without drawing.

Researcher: Powerful! So can you teach integer operations using C-I-S model to your pupils when you start to teach?

Learner #5: 29 Yeah… I'm ready and eager to.

Discussion

It is instructive to note that each learner made positive gains in their performance from the pre-test to the post-test, with a mean gain of 47.82%. A large amount of the change witnessed is explained by Kamina and Iyer (2009). They found that hands-on experiences make learners appreciate "how" numerical symbols and abstract comparisons work. The increased use of concrete manipulatives at the initial stage influenced this gain trajectory. As Jones and Tiller (2017) revealed, using hands-on, concrete manipulatives can lead to higher retention rates and a more positive student attitude toward mathematics in particular and education in general. In their analogous study, they contend that using C-I_S allows students to make associations from one stage of the process to the next. Manipulatives can be particularly effective in further developing conceptual understanding in mathematics.

The new mathematics curriculum (MoE-G, 2019) strongly advises against teaching mathematics without using manipulatives, especially in the primary grades. Using manipulatives is recommended in theory (Bruner, 1966; Drummond, 2021) and research related to classroom learning (Akayuure et al., 2016). Drummond (2021) and Jones and Tiller (2017) found that using manipulatives in the classroom increased learners' performance tasks, further supporting the performance gain. The outstanding performance brought about by manipulative use means that teaching using Bruner's 3-tier model improves performance and deepens understanding of a specific mathematical concept. Bruner (1966) proposed the C-I-S (Concrete-Iconic-Symbolic) method of teaching, which starts with using concrete materials and progresses to using symbols through pictures. In a related study utilising a transition from concrete-representational-abstract transition, Jones and Tiller (2017) found a profound increment in pupils' conceptual knowledge. This study, therefore, reaffirms this renowned theory.

In terms of curriculum direction, Jones and Tiller (2017) noted that there has been a focus on inclusive education in recent years. Inclusive classrooms are frequently praised for diverse students' academic abilities and as an excellent way for them to learn about respect, and appreciate one another's learning styles. However, inclusive classrooms can leave teachers unsure of how to approach each student's unique learning needs. A shift toward the C-I-S mode of instruction would provide the necessary balance to meet the learning needs of diverse learners. Hands-on interaction with concrete manipulatives allows students of all mathematical levels to begin instruction on an equal playing field.

If Bruner's 3-tier theory is followed correctly, the concrete and iconic stages are a foundation, propelling learners to more excellent proficiency in algebra and related fields. As noted by Konyalioglu et al. (2005), images, pictures, and diagrams are very useful mediators between concrete materials and abstract representations, broadening learners' baseline knowledge. Bobek and Tversky (2016) found that creating a visual explanation was superior to verbal explanations for complex systems in science, corroborating this study. The study's findings thus support visual explanations as practical learning tools.

As Bruner (1966) intimated, some pictures give vital information to thin the understanding gap. It means here that the bigger the understanding gap, the more complex the understanding, and the thinner the gap, the less complicated the understanding. Therefore, using and applying a rule or the following procedure to solve a task becomes less frustrating using pictures. Additionally, manipulatives ease learning by giving an extra channel for transmitting the application of the materials learnt to the physical world.

One commonality among all the five learners interviewed was that the entire intervention lessons were engaging. The appraisal by interviewees means that if mathematics concepts are taught using appropriate modelling, especially the one espoused in this study framework, they will arouse learners' interest. All these five learners were unmotivated learners, but through meaningful modelling of concepts that hitherto confused them, they became activated learners.

Another common phenomenon extracted from the interview was that all interviewees agreed that using models and modelling integer concepts in line with C-I-S made it easy to understand and apply the knowledge to new and different contexts. As Cabahug (2012) opined, students must learn mathematics with understanding. Understanding and application are the two most important pillars that underpin the Ghanaian basic school mathematics curriculum assessment, with respective weightings of 30% and 70% (MoESS, 2007a, 2007b). It was clear from the interview dialogue that all the learners had developed high self-efficacy toward teaching integers and were eager to teach mathematics during microteaching and beyond. The learners' healthy spirit runs counter to earlier ones before the intervention using C-I-S modelling. The uniqueness of the intervention aligned with Aduko (2016) when he suggested that concrete scaffolding should precede every concept development and progressively wean learners from it. To him, appropriate conceptual scaffolding propels students to become independent, confident learners.

Literature provided by Currell (2021), Drummond (2021) and Cabahug (2012) points out that irrespective of one's age, the mind configures new materials through three qualitative stages, namely, C-I-S, which is the basis of Bruner's (1966) theory. According to Drummond (2021), the more frequently this C-I-S is followed in concept presentation, the more effective learning becomes. Thus, the zealous and confident manner in which learners are willing to apply the knowledge acquired through this intervention affirms Currell's (2021), Drummond's (2021) and Cabahug's (2012) literature. It follows that learners' willingness to apply knowledge learnt increases when the concept is understood correctly (see lines 16, 22, and 24). The interviewees not only demonstrated their ability to explain how to perform certain operations clearly that appeared difficult to them prior to the intervention but corroborated their understanding prowess by the high scores registered in the post-test.

Bruner (1966) theorised that learners could carry out abstract tasks fairly well when new concepts are presented following this 3-tier theory of C-I-S. This theory is confirmed in this study. An instance of this assertion is corroborated by (Learner #5 in line 28) that "if you understand how it works, you can do it mentally and get the answer without drawing" (Learner #5).

Conclusion

This research was undertaken to improve learners' understanding of integers by employing Bruner's 3-tier-theory of concept presentation. The research design was action research. The researchers purposefully selected 82 teacher trainees in their first year of semester one at Gbewaa College of Education, Pusiga. Data were gathered using pre-test, post-test, and interviews. The study confirmed that one of the sources of learners' difficulty is the confusion between the unary and dual meaning of the minus sign. Another source of difficulty is low self-efficacy toward mathematics.

The post-test scores demonstrated quantitative gains in the learners' conceptual knowledge. Lesson presentations in tandem with the C-I-S construct resulted in qualitatively significant, meaningful, and connected learning. The 3-tier model simplifies knowledge comprehension and application. Moreover, learners' opinions on the C-I-S construct were primarily positive because it piqued their interest and motivated unmotivated students.

There was a substantial change in the conceptual knowledge of learners who participated in the intervention. The intervention utilised Bruner's 3-tier theory—presenting lessons in line with the C-I-S construct alleviated learners' difficulty with integer operations like the confusion between the dual and unary meaning of the minus sign and low self-efficacy, among others.

In addition, Bruner's 3-tier theory positively impacted learners' conceptual knowledge of integer operations. Similarly, there is a need for more college mathematics tutors to use the C-I-S mode of lesson delivery.

Recommendations

The study recommends that departmental college-based workshops for subject tutors be organized to orient them on the C-I-S style of lesson presentation and allow them to buy into the idea. These departmental, college-based workshops would allow them to integrate it into their regular lesson delivery. Aside from lesson integration, subject tutors should deliberately model and deliver lessons for teacher candidates to observe.

Similar workshops could be extended to mentors and lead mentors, who would be responsible for the upbringing of trainees for most of their micro-teaching phase.

Supervising tutors should explicitly remind and encourage teacher trainees to use C-I-S modes of instruction during micro-teaching sessions.

Aside from lesson integration, subject tutors could model and deliver a lesson (or lessons) for teacher trainees to observe.

Similarly, similar workshops could be extended to mentors and lead mentors, who would be in charge of trainees' upbringing for most of their micro-teaching phase.

During micro-teaching sessions, supervising tutors should remind and encourage teacher trainees to use C-I-S modes of instruction.

Limitations

A few validity concerns are raised in the literature concerning action research, such as controlling confounding variables. From a contextual standpoint, attempts were made by the tutor/researcher to randomize the group as well as lessen the Hawthorne effect. Also, of the over forty (40) colleges of education in Ghana, only one college was involved in this study, making the sample size very small and limiting the extent to which findings could be generalized to the entire population. Further, the insufficient duration of this study also creates an obvious limitation. The very few lessons in this study definitely cannot achieve a compelling result. One of the limitations in this study also relates to the non-existence of information regarding Bruner's 3-tier theory in the Ghanaian mathematics education research area. The researchers were unable to draw from more local examples and knowledge.

Authorship Contribution Statement

Aduko: Conceptualization, design, securing funding, data acquisition. Armah: Editing/reviewing, supervision, statistical analysis, material support.

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