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Conceptions of Mathematics Teacher Educators Depicting Essential Mathematics Teacher Educator Knowledge

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Abstract: Research into knowledge which mathematics teachers require to teach abounds. There is also mounting interest among mathematics teacher education researchers to characterize mathematics teacher educator knowledge (MTEK). However, there is a generic dearth of studies focusing on conceptions of mathematics teacher educators (MTE) regarding MTEK. This article is a product of a qualitative case study underscoring teacher educator conceptions in that regard and the investigation involved two MTE who were practicing in a university. The research site was conveniently chosen, and participants were intentionally selected to respond to interview questions which elicited espoused views. Narrative analysis was used through exploration and subsequent interpretation of transcripts which aligned with questions posed. Analyses suggested a complexity to exhaustively categorize the MTEK necessary for MTE to train mathematics teachers. Notwithstanding, MTE believed that MTEK should include understanding of research in mathematics teacher education and teaching, mathematics teacher knowledge, and MTE professional development. Additionally, the findings suggested that MTE acquire mathematics teacher educator knowledge through postgraduate studies, on the job practice, mentorship, and participation in professional development activities. Research in other contexts is recommended to identify mathematics teacher educators' understandings of MTEK and how that knowledge should be acquired.

Keywords: *Mathematics teacher educator conceptions, mathematics teacher educator knowledge, mathematics teacher knowledge, mathematics teacher quality.*

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Introduction

One of the significant factors in the process of teaching and learning mathematics is that of mathematics teacher quality (MTQ). Connected to MTQ is the nature of understanding demonstrated of the mathematics taught by mathematics teachers. In this regard, the field of mathematics education is replete with research literature on mathematical understandings of both student teachers and teachers of mathematics (Dede & Soybaş, 2011; Evangelidou et al., 2004; Malambo, 2020, 2021; Malambo et al., 2018, 2019). Likewise, teacher knowledge frameworks and those for examining mathematics teacher knowledge have been developed historically (Ball et al., 2008; Peng & Luo, 2009; Shulman, 1986). The desire to enhance teacher quality has led governments in some countries to resort to regulatory undertakings (Australian Institute for Teaching and School Leadership, 2015; Zambia Ministry of Education [MOE], 2013; National Council for the Accreditation of Teacher Education, 2008). In the current author's country of origin, it is now a legal requirement for practicing teachers to be licensed by a regulatory organization (MOE, 2013). At the same time, teacher training departments in universities and colleges of education are obliged to have the mathematics education programs and courses on offer accredited with the higher education authority. In addition to the necessity for courses approval, mathematics teacher educators (MTE) are required to provide their qualifications and areas of specialization during the application process for programs' accreditation (MOE, 2013). These developments are consistent with the view that MTQ is inextricably linked to mathematics teacher education programs and quality of MTE.

Accreditation of mathematics teacher education programs and provision of MTE's qualifications to regulatory organizations could give the impression that there is consensus regarding the mathematics teacher educator knowledge (MTEK) which MTE should have. Nevertheless, the nature of knowledge which mathematics teacher educators should possess for effective training of mathematics teachers is neither specified by the regulatory organization nor exhaustively

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articulated in research literature. While there is growing recognition internationally in terms of the necessary teacher knowledge for mathematics teachers, there is no research evidence of consensus regarding the MTEK. Moreover, the source of teacher educator knowledge is not documented. Without doubt, mathematics teacher education is influenced by several factors such as systems of governance in different countries of the world. Furthermore, whether mathematics teacher education curricula are centralized or decentralized, there are variations in structures of mathematics teacher education systems across countries (Belbase, 2019; Tatto & Senk, 2011). There are also disparities in what is deemed to be essential MTEK required for training teachers of mathematics across countries. The contents of mathematics teacher education courses are equally not standardized across countries and institutions of higher learning.

Although researchers have presented accounts about teacher educator self-study (Schuck, 2002), MTEK has not received much attention in research (Beswick & Goos, 2018; Oates et al., 2021). Researchers have mostly focused on the understandings expected of mathematics teachers with little effort exerted towards identification of the knowledge which should be acquired by teacher educators to effectively perform the role of training teachers. The experiences and beliefs of MTE are rarely considered, for example, what they go through and believe to be essential MTEK. Anecdotal testimonies indicate that it is normally assumed that teacher trainers automatically acquire necessary knowledge through postgraduate studies and that they can perform the functions expected of them upon completion of such studies. This could suggest that mathematics teacher educators do not require opportunities for improvement and yet the work of teacher educators involves "thoughtfully engaging with practice beyond the technical" (Loughran, 2014, p. 274). Of course, it is easy to assume that mathematics teacher educators have the capacity to train teachers and understand the knowledge required for performance of that role. However, reality in teacher training institutions may suggest a different scenario. Therefore, it is important to not only explore what mathematics teacher educators know of teacher training in general, but also what they know about necessary MTEK.

Chapman (2008) contends that it is important to investigate mathematics teacher educators' teaching approaches and activities they provide for student teachers. Such investigations could provide a glimpse of what teacher educators consider as essential knowledge for their work. The investigations could also partially assist in understanding the effectiveness of teacher educators including strategies that are effective or those requiring modification in mathematics teacher preparation. Beswick and Goos (2018) propose the necessity to establish what mathematics teacher educators should know and how different MTEK is from teacher knowledge and that of mathematicians. The two authors posit that it is important as it is for mathematics teachers to understand the beliefs that underlie the activities of MTE. Furthermore, they assert that in this era it is appropriate to "consider the ways in which MTE become knowledgeable and to develop research-based models and processes that move the development of expertise beyond incidental on-the-job acquisition that has prevailed in many contexts" (Beswick & Goos, 2018, p. 418).

The preceding considerations including rarity of research around MTEK (Beswick & Goos, 2018) are a motivation for research which can provide a basis for characterization of MTEK. In this article, conceptions regarding what practicing MTE perceive as necessary knowledge for training mathematics teachers are provided. The conceptions are given in response to the following questions: (1) What kind of knowledge do mathematics teacher educators espouse as necessary MTEK for effective training of mathematics teachers? (2) What are the sources of mathematics teacher educator knowledge? (3) What challenges do mathematics teacher educators encounter when training mathematics teachers? Methodological aspects relating to this article are hereafter presented.

Methodology

Research Design & Philosophical Orientation

This article is based on a qualitative case study whose intention was not generalization of the findings (Nieuwenhuis, 2014a), but acquisition of in-depth understanding of necessary MTEK through the lenses of MTE. The ontological perspective of the investigator was that truth does not exist independent of the researcher while an epistemological assumption crafted was that reality can be known by exploring peoples' experiences concerning phenomenon. Although bias is likely in people's experiences, and narratives, this assumption was preferred because of the researcher's belief that "experiences and voices of the respondents are mediums through which we explore and understand reality" (Nieuwenhuis, 2014b, p.55).

Sample and Data Collection

Two MTE who were practicing at a conveniently chosen university were the sample. The participants were intentionally chosen due to their immense experience in mathematics teacher education. Both participants studied mathematics at first and master's degree level and their doctoral degrees were in mathematics education. The two MTE had previously taught mathematics in secondary schools before their appointment as mathematics teacher educators in a university. Furthermore, one of the participants was at the rank of professor and had been training teachers of mathematics in university for more than 10 years. Moreover, this participant was an accomplished researcher with several published research articles in mathematics education. Likewise, the second participant, although not at the professorial rank, had taught university mathematics and trained teachers of mathematics for a long time.

Researching at a single teacher education institution mitigated the likely conflicting influences of variations in the structures of teacher education institutions across the country. MTE practicing in a continent different from that of the researcher were sampled primarily to reduce the possible risks identified in literature (Beswick & Goos, 2018) and to facilitate acquisition of lessons from a diverse context. Beswick and Goos (2018) reveal that MTEK has previously been researched by MTE through several self-studies. These authors argue that self-studies can bring into question the "trustworthiness of findings and, second, there is a danger that important questions will not be asked" (p. 423).

The sample participated in face-to-face individual interviews which were exclusively developed for the study. Interview questions focused on the kind of knowledge which the MTE deemed necessary for training mathematics schoolteachers and the possible sources of that knowledge. Concurrently, views of the MTE concerning aspects which should be emphasized as teachers of mathematics get trained were elicited. Furthermore, challenges encountered by the MTE in their task of training teachers of mathematics were investigated. In these endeavors, MTE were requested to provide justifications for their viewpoints.

Analyzing of Data

Narrative analysis was utilized because the study was about narratives which were a response to open-ended interview questions. This kind of analysis involved the search for narrative strings, threads, and themes (Nieuwenhuis, 2014c). Interviews were audio recorded and thereafter transcribed for analysis and interpretations. The researcher read the transcripts several times, and subsequently, responses were organized around the common questions posed to the two respondents. During that process, the transcripts were also explored for common responses to similar questions. A comparative strategy to categorize excerpts that were relevant to common questions posed was used. Consideration was made to determine whether there were instances when varied answers were given by respondents to the same questions. Manual coding was conducted, and categories developed. Resultant categories were reflected on by the researcher through the eyes of the participants to ensure correct interpretation. Similarly, interpretations were conducted through the lenses of the researcher, and participants. These processes which were undergone are consistent with the views of Merriam (2009) who contends that data analysis relates to making meaning in the context of what people say including what the researcher has seen and read. Deductive ideas adapted from the works of Dreher et al. (2018), Murray et al. (2017), and Zaskis and Leikin (2010) were equally utilized to analyze the data. The next section provides narratives of the sampled MTE.

Findings/Results

Results presented in this section include representative excerpts of questions posed, and those of respective responses by the mathematics teacher educators. To promote anonymity, the pseudonyms; Kamagoi and Bezil are used. The thrust of interview questions was determination of the knowledge which the participants considered necessary to be held by mathematics teacher educators for purposes of training teachers of mathematics. Selected excerpts and corresponding analyses which are consistent with the focus of the present article are now presented below:

Interviewer (I): What kind of knowledge do you require to effectively train mathematics teachers as a mathematics teacher educator?

Kamagoi (K): Ah, that is a good question, a sort of new line of research too and that is a loaded question. I need to be well versed with the research around mathematics teaching and mathematics teacher education. I need to know my student teachers; I need to get to know them to see how they learn best and then figure out how to teach in a way that will reach them.

Kamagoi's views suggest that MTE should be acquainted with research in the context of mathematics teacher education and teaching. By implication, she thought that research findings in the specified spheres ought to inform mathematics teacher educators' practice. Kamagoi also emphasized the necessity for MTE to have generic knowledge of how mathematics student teachers learn. That knowledge could assist teacher educators to devise ways of teaching the student teachers. The idea was that understanding of the student teachers one is training can assist in tailoring what ought to be taught including what to emphasize. Without specifying the kind of knowledge, Kamagoi speculated the need for MTE such as herself to have 'enough knowledge' to facilitate student teachers' contextualization of what they are taught in training institutions:

K: I might need to have enough knowledge to help the pre-service teachers contextualize the information that they are getting from what they are learning. I do not know exactly how to articulate that, but I guess in addition to knowledge of research and best practices for teaching mathematics and how students learn mathematics it is also understanding like school, culture, and schooling and how that works and then also how the larger society and other things influence teaching and learning.

Kamagoi believed that besides knowledge of research, MTE should be abreast with the 'best practices' in the context of teaching mathematics. She posited that mathematics teacher educators should understand what goes on in schools, inclusive of cultural issues and how society and varied other factors do influence the teaching and learning of

mathematics. Her views regarding the specific kind of knowledge a mathematics teacher educator requires from research to function are reflected in the ensuing discussion:

I: Specify the kind of knowledge from research that you require to function as a mathematics teacher educator.

K: OK, I do not have the actual knowledge now [pauses]. I'm not sure of the exact answer like knowledge of students, knowledge of the content [pauses]. I do not know what, maybe let me add [pauses]. Sorry, for the mathematics student teachers, yes. When you are tackling the idea of the knowledge that they need, yes, they should be able to explain concepts that they would teach etc.

The preceding extract suggests that Kamagoi was unable to pointedly spell out the kind of knowledge from research which is necessary for a mathematics teacher educator to train teachers of mathematics. It seemed easier for her to give hints of the knowledge which mathematics student teachers are supposed to acquire. This scenario was not surprising because research on teacher knowledge abounds while knowledge that mathematics teacher educators should possess for effective performance of the role of training teachers is not specified in research. Gradually, Kamagoi explained that it is necessary for mathematics teacher educators to have in-depth understanding of the same knowledge that mathematics teachers in schools are required to have. She reiterated that a mathematics teacher educator requires to understand the school system and how mathematics teacher educators' knowledge could be applied in the school system:

I: Your view is that what mathematics student teachers should know for teaching purposes, you must already know it as a mathematics teacher educator?

K: Yes, and that is like at the bottom, that is like the message baseline. Then in addition to that, I need to have a good understanding and knowledge of the schools, the schooling system, how it works, and how we take this knowledge that we have and then think about it realistically, like what is it that you can do. What are some constraints? That is kind of a little bit for mathematics teacher educators to have a good grasp, because I cannot go and tell student teachers a whole bunch of stuff that they can do without being realistic of what really can be accomplished in a classroom. I do not want to say that I like each mathematics teacher educator must have had these many years of mathematics teaching experience in like a middle school or whatever you know, which I do have that experience, but I as a mathematics teacher educator needs to have a good grasp of the how of teaching and learning mathematics, like in the schools. I do not know how to explain this differently.

Kamagoi intimated that in addition to understanding the school system, a mathematics teacher educator must have school teaching experience in mathematics. Furthermore, she contended that MTE ought to understand pedagogic practices that enhance learning of mathematics by school pupils. She advocated for MTE to understand what was deemed as the 'how of teaching and learning mathematics'. Arguably, this view suggests that MTE should not only understand the mathematics taught at school level but should likewise be able to teach student teachers ways that would enhance learners' understanding of the mathematics taught. Another perspective shared by Kamagoi was that mathematics teacher educators should have capacity to help student students to adapt knowledge acquired from research to school mathematics classrooms:

K: Like we talk about what we know from research about how students learn mathematics, right, but in terms of the implementation of that knowledge, I need to have a good grasp of how one would do that in a class of 30 students knowing that you only have like 50 minutes a day. That you know that the student teachers must take whatever tests, that you know that they may be going through a specific type of trauma that you know. And I'm not going to be the super expert on that, that is the big role that the school-based teacher educators play.

While recognizing the significance of MTE having capacity to adapt research findings to classroom teaching, Kamagoi claimed that school-based mentors have a role to play in that regard as well. Both Kamagoi and Bezil were of the generic view that MTE should understand mathematics for them to effectively train teachers of mathematics. The discussion with Bezil concerning the kind of mathematical knowledge which mathematics teacher trainers must understand went as follows:

I: What do you think should constitute the mathematical knowledge of a mathematics teacher trainer such as yourself?

B (Bezil): I think that a master's degree in mathematics is a good place to start because I think that you should have a good understanding of all the undergraduate mathematics and then a little bit above. So, a deep understanding of mathematics past the bachelor's level for someone that is focusing on mathematics. If you are teaching just the education side, I think just having the undergraduate knowledge of mathematics.

Bezil could not specify the mathematical knowledge which a mathematics teacher trainer must have. Instead, she indicated the necessary teacher educator qualifications and advocated that mathematics teacher trainers must understand, for example, undergraduate mathematics. This perspective seems to neglect the fact that mathematics courses at bachelor's or postgraduate degree levels are not uniform in terms of names and content in different universities of the world. Bezil was consequently requested to be explicit:

I: I want you to be precise in terms of the mathematical knowledge itself. As a teacher trainer, what mathematical knowledge enables you to train the mathematics student teachers?

B: I think getting into some of the abstract mathematics courses so, uh, like uhm number theory. Um analysis, Uh, the abstract algebra and things like that. Yes, now this content knowledge is university based it is not school based, yeah.

Whereas Bezil attempted to give names of mathematics fields such as number theory, analysis, and abstract algebra, her response was still short of a specific description of mathematical knowledge for mathematics teacher educators. Nevertheless, Kamagoi and Bezil described their general and common understanding in terms of the level of mathematics required by mathematics teacher educators to function. An extract from the conversation held with Bezil typifies an overview of that understanding:

B: As a mathematics teacher trainer, you should also have knowledge of the mathematics that is taught at each level that you are training the teachers for so that you can make connections between the more complex mathematics and what is behind those. And then how that is applied in a middle or high school classroom. You really must have a range of mathematical knowledge, because if a teacher educator only knows the university-based mathematics, they might not really understand how that university-based mathematics is applicable to a 12-year-old. And if you only have a knowledge basis that is at the middle or high school level, then you may not be able to make connections between different concepts. You may not understand why things work the way they do. You may not be able to explain or anticipate misconceptions. So, you really must go from the lowest level to a little bit past undergraduate level. You should know what a learner is expected to know yes way lying beyond quite that. You must know both what the student teachers you are training are going to teach and the university mathematics.

The teacher educators' view was that in addition to understanding 'university-based mathematics', teacher trainers need to understand the mathematics taught at school level. The rationale is to facilitate both the teacher educators' and trainee teachers' capacity to make connections between university and school mathematics. In addition, Bezil was of the view that mathematics teacher educators ought to possess comprehensive knowledge of the contents of the courses taught. She attempted to explain how knowledge of the connections between advanced mathematics and school mathematics influenced her teaching:

I: As a teacher trainer in this university, in which way does your knowledge of the connections between advanced mathematics and school mathematics directly influence your teaching?

B: Because I have taken some of the advanced mathematics courses like in the master stream, not just at the undergraduate level, I feel like I have kind of a more complete view of how everything is connected to each other. And somehow even things as simple as even and odd numbers. How expressive you can be like with number theory and writing it out, which is, I think something that when you are more of a novice in mathematics, you do not necessarily get an appreciation for how to take these very simple topics that seem very simple but like expressing them mathematically or proving simple theorems that we take for granted can be complicated.

Bezil admitted that there was a way in which her knowledge of connections was positively influencing her teaching. Knowledge of connections enabled her to appreciate mathematics concepts which somehow contributed to her effort to ensure that the student teachers acquired similar understanding. A hypothetical scenario was presented to the interviewees to articulate the kind of knowledge that they would require to train teachers of mathematics if they were novice mathematics teacher educators. The views of Kamagoi are encapsulated in the excerpt below:

K: Here is the thing. I need to have knowledge like how students learn mathematics with understanding and what are some ways that I should teach the student teachers the mathematics. Let's talk about middle school mathematics teacher preparation. I need to have [pauses]. I need to have extensive knowledge of everything that my pre-service teachers need to know. I need to already know everything that I want to teach them. I already need to know that well. I need to know the mathematics. I need to know how students learn mathematics.

The views in this excerpt about novice MTE corroborate the views of Kamagoi and Bezil in the context of knowledge required by experienced MTE. The excerpt suggests that novice MTE ought to understand the mathematics taught at school level as well as the pedagogic aspects that would enable teachers to effectively teach mathematics. The two teacher educators believed that MTE cannot teach student teachers what they do not understand themselves. The idea of understanding how students learn mathematics suggests that MTE should understand the theoretical bases of effective teaching and learning of mathematics.

Perceived sources of the essential knowledge for mathematics teacher educators were not explicitly explained by interviewees. The teacher educators, however, hinted what they espoused as sources of knowledge that assisted them to perform as mathematics teacher educators. Both Kamagoi and Bezil acknowledged implicitly that teacher educator knowledge was accessed in different ways and progressively. Kamagoi disclosed of having first been a subject schoolteacher, and then a researcher. She explained that such phases in her professional life were necessary for acquisition of knowledge to train teachers of mathematics. Furthermore, she insinuated that documents written by other scholars, and postgraduate studies were, to her, other sources of mathematics teacher educator knowledge. Kamagoi

affirmed that the doctoral program in mathematics education at her university contributed to acquisition of the knowledge required by mathematics teacher educators to train teachers of mathematics. Although the said doctoral program had important links to the job of a mathematics teacher educator, Kamagoi pointed out that the program was not exclusively intended for mathematics teacher educators.

Similarly, Bezil explained the source of the knowledge which she used when training teachers of mathematics in the context of her career pathway. Her career stretched from being a high school mathematics teacher after acquiring a bachelors' degree in mathematics. Then a master's degree in mathematics was pursued after which she taught mathematics content courses in the university. Her PhD was in mathematics education and concurrently she acquired practical experience in school set ups. The following excerpt confirms:

B: I first got my undergraduate degree in mathematics education and taught high school mathematics. And then I came back to do a master's degree in mathematics. I got the master's degree in mathematics, and taught mathematics courses in the mathematics department and thereafter I got my PhD in mathematics education. I went back and forth between the two departments and while I was doing my PhD in mathematics education, I did a lot of supervision in the middle school. I got to see even more of what the mathematics that is happening in the middle school classrooms was like as well.

Bezil's teacher educator knowledge seems to have been acquired through teaching experience at school level, postgraduate studies, on the job practice, and observing what goes on in school mathematics classrooms. Arguably, knowledge acquired in mathematics content, mathematics education, and school mathematics classrooms was perceived to have contributed to Bezil's capacity to train teachers of mathematics. The probing continued as follows:

I: As a mathematics teacher educator, there is no institution specifically meant for you to train as a teacher trainer. You may get your masters, and your PhD, but usually you focus on a certain aspect, right?

B: Yeah, that is a great question. I felt very prepared with the mathematics knowledge that I have because I had my undergraduate mathematics and my master's level. I felt very prepared for that, but when I started the PhD program and started working with the pre-service teachers, I did feel a little out of my depth on applying that mathematics knowledge and helping pre-service teachers on how to teach it. I knew the mathematics, I was very comfortable with that, and I had taught in a classroom, but I had not taught teachers how to teach in a classroom. So, when I started my PhD, I felt very inexperienced, but I was taking classes on training teachers, preparing teachers for K12 and so I had a lot of strategies for how to teach teachers and how to help them make connections. And then I worked with other people in the department that had experience to learn from them and what they knew. And then I went out in the schools too and worked with the teachers in the classrooms to see what they wished that the pre-service teachers knew and what they wished that we would be making connections of. And so, I think that it was just a matter of learning from others who had already done a lot of that learning not just the university classes on training teachers, but also from teachers in the field. What they wish they had known earlier and things that they knew.

Despite lack of university programs specifically designed to prepare trainers of mathematics teachers, the idea of experienced mathematics teacher educators providing mentorship to novice mathematics teacher educators was emphasized. Bezil recommended the necessity for the mathematics teacher educators to learn from experiences of practicing schoolteachers and for them to acquire practical knowledge by operating in actual school classrooms. Furthermore, the need for courses that are exclusively designed to train prospective mathematics teacher trainers was emphasized:

I: What would you recommend then from your experience as a mathematics teacher trainer?

B: There should be courses tailored towards training mathematics teacher educators.

The ideas from Bezil and those championed by Kamagoi corroborated and suggested that prospective mathematics teacher educators must acquire understanding of mathematics content and knowledge regarding how to help mathematics student teachers to teach. Bezil asserted that knowledge of teacher trainers is attained over time. She provided an example of herself as having experientially and progressively acquired the relevant knowledge. While acquisition of a qualification is necessary, it is not sufficient as experience developed over time is required:

I: So, to a mathematics teacher trainer experience is critical?

B: Yes, but we should allow services to begin. Something that really helps is observation of experienced university teachers teaching. I observed Dr [name of lecturer] a mathematics instructor who runs her class very well and is knowledgeable of the subject matter. Instead of teaching her classes just as mathematics classes, she really tried to connect to the education stuff, and I observed her for two semesters teaching courses. That way I could, you know, get a little bit more knowledge and experience, and I observed [name of professor] and [name of professor] teach a bunch of classes as well so I could get a little bit more experience. And I have observed even schoolteachers in the classrooms. Sometimes watching them because they have built up such a wealth of knowledge over the years that they have been teaching. They will make connections that then I see like, oh, ok, ok, I need to make sure that I make that connection as well. So, mathematics teacher trainers must not shy from learning that I know this thing so there is nothing I can learn

from these teachers. I'm always learning new things and sometimes I'm even learning new things from the pre-service teachers because sometimes they will just express something in a way that I hadn't thought about it before. So, I think you must be open to new knowledge from multiple sources.

This extract suggests that teacher educators should not wait for experience to begin operating. In line with this understanding, Bezil posited that her knowledge was accessed from multiple sources, for example, she had mentorship from experienced mathematics teacher trainers. She got capacitated and acquired valuable lessons by observing experienced teacher educators and mathematics schoolteachers regarding how they conducted their lessons. Likewise, Bezil acquired knowledge by listening to her student teachers.

The teacher educators commented on the challenges which they had encountered in their work of training mathematics teachers. Both respondents acknowledged having experienced challenges which, however, were of different orientation. Kamagoi's challenges are explained in the following extract:

K: Some challenges I have had? I'm trying to think [pauses]. I know I had to like [pauses], the several challenges [pauses], so I'm trying to reflect on like what are the different things. It depends on the population, for example, when I was teaching elementary mathematics teachers a big challenge that I think is common with those, of course, is the elementary teachers [pauses] there are a lot of elementary teachers who don't like mathematics, or they have some sort of type of mathematics anxiety. For example, they know they want to be elementary teachers, but it is like their least favorite subject to teach is mathematics, you know they are afraid of mathematics. So that was a big challenge when I was teaching those courses and so trying to figure out ways in which I would make them feel comfortable in the course and so that was a challenge because a lot of times a lot of them would come in already, like with a negative attitude.

Kamagoi was hesitant initially to point out the challenges she had experienced when training of teachers of mathematics. She eventually mentioned the challenges but did not associate those challenges with a lack of knowledge required for MTE. Kamagoi attributed the challenges she experienced to the category and attitude of the student teachers taught. She motivated her considered view as follows:

K: You know how I talked about the elementary pre-service teachers, how a lot of them come in with some negative attitude or anything like that. They are a different kind of population from secondary mathematics teachers. But for those that want to teach high school mathematics some challenges that I had with that population is that they come in thinking that I'm only teaching content and mathematics is the most important thing and I do not care about anything else just to find out no, but you are teaching teenagers and there is like a whole bunch of stuff happening there in their lives, you know. So, I think the challenge with them is to like to help them see that, yes, mathematics content is important but teaching the mathematics content is not just about the content, so there is a whole bunch of other things.

Kamagoi emphasized the importance of having a clear understanding of the level at which the teachers being trained will be teaching. Her explanation suggested a need for knowledge, on the part of mathematics teacher educators, that would enable them to re-orient the mindset of some of the categories of student teachers as well as to ultimately motivate them. The discussion with Bezil was conducted in the context of what she considered to be her professional deficiencies:

I: In which areas do you, as a mathematics teacher educator, feel deficient professionally?

B: I always feel like I'm learning more and more about different ways of expressing mathematical ideas in the classroom. So, it is not that I could point out something specific that I'm deficient in, but I am always trying to look for new ways to visually represent things in new ways. To explain a concept to students and talking to pre-service teachers about what they should be doing in the classroom. I'm always just trying to increase my toolbox of how you expect to increase your toolbox of knowledge and not necessarily the mathematical knowledge, but the ways of expressing that knowledge.

Bezil's perspective highlighted a requirement for continued learning and improvement on the part of a mathematics teacher trainer. Instead of considering what was not known as a deficiency, she considered such as an opportunity for learning and improvement in the context of helping student teachers of mathematics. This perspective is at variance with a conventional view that teacher trainers are adequately knowledgeable and consequently do not require opportunities for improvement in their role of teacher training. Bezil articulated examples of how she empowered herself about teacher educator knowledge:

I: How do you empower yourself in terms of teacher educator knowledge?

B: I just try to find resources, to find different visual representations, different tools for expressing things up, things like that.

Kamagoi and Bezil gave additional considerations that are inevitable in the process of training teachers of mathematics. A representative conversation with Kamagoi is presented below:

I: What additional considerations do you need to make as you train teachers of mathematics?

K: Additional considerations! So, really understanding what you are teaching while you are teaching it. I think those are types of things that mathematics teacher educators need to have as a framework. Uh, understanding the students that you are teaching.

Kamagoi re-emphasized the importance of considering the subject matter taught to student teachers and the necessity to acquire understanding of the student teachers taught.

Discussion

The discussion in this section is about conceptions of MTE relative to the knowledge which they considered to be necessary for performance of the task of training mathematics teachers. The narratives depicted a perspective that mathematics teacher educators should be knowledgeable in research which is consistent with mathematics teacher education and the teaching of mathematics. This conception presumes teacher educators' understanding of trends in research about mathematics teacher education and teaching, and how to use research findings when training teachers and in teaching mathematics. These facets suggest implicitly that MTE should have the capacity to conduct research in mathematics education. Capacity in that context is meaningful if it leads to learning opportunities for student teachers through guided activities in which research evidence assists to resolve challenges in teaching mathematics.

Another conception derived from the narratives is that MTE require knowledge concerning what teachers of mathematics are required to understand for effective teaching of mathematics. The conceptions of teacher educators corroborated the perspective of Beswick and Chapman (2012) who asserted that MTEK involves the knowledge which mathematics teachers use to teach. Since MTE are expected to teach prospective teachers how to teach mathematics, they should for example have ability to make appropriate pedagogic decisions. This perspective aligns with Murray et al.'s (2017) view that an aspect which influences pedagogical choices is knowledge of mathematics connections. Arguably, MTE need to understand issues of mathematics connections between advanced and school mathematics. That understanding presupposes in-depth knowledge of the subject matter taught at school level inclusive of that of advanced mathematics and the appropriate pedagogies. MTE ought to understand the mathematics subject matter which student teachers are required to acquire in-depth understanding therein. Thus, mathematics teacher educators should have knowledge of what they expect their student teachers to know. This view corresponds with an understanding that teacher educators can only illustrate to student teachers how to teach mathematics if they understand the subject matter and how to teach it. Simultaneously, mathematics teacher educators should understand, among others, the subject matter which is beyond what is expected to be understood by their trainees. The notion is that MTE should have deeper understanding of the subject matter than their student teachers just like the expectation concerning teachers to understand mathematics beyond the levels they teach pupils. Teacher educators' understanding is expected not to solely be in terms of the 'quantity of knowledge' but should equally be about the nature and depth of understanding. The understanding of MTE should be of a nature that can enable them to guide prospective teachers to unpack subject matter for pupils' conceptual understanding. These ideas corroborate Loughran's (2014) assertion that the work of teacher educators as it relates to engagement with practice goes beyond the technical.

Mathematics teacher educators are expected to have unique knowledge in the context of their role of training teachers of mathematics. This corroborates the argument in previous research that MTE "need a particular form of knowledge about teaching mathematics and they need to hold this knowledge in a way that is rather different from the way that teachers know it" (Beswick & Goos, 2018, p. 418). An example is that MTE require a repertoire of strategies to employ when guiding student teachers how to teach school mathematics concepts (Malambo, 2021). As intimated earlier, MTE should have in-depth understanding of the school mathematics concepts and then be capable of teaching those concepts if they are to effectively guide student teachers. Of course, teaching prospective teachers school mathematics concepts and teaching them how to teach those concepts are two different things. Besides, mathematics teacher educators' knowledge must include understanding of the connectivity of mathematics concepts (Malambo, 2021), and how to connect different concepts in the process of teaching. Furthermore, MTE should understand their student teachers and how the student teachers learn. This is like the obligation placed on teachers of mathematics where they ought to understand their pupils and how pupils learn. The view that MTE ought to know how student teachers learn could refer to, for example, how the student teachers learn advanced mathematics, school mathematics, or how the student teachers learn how to teach specific school mathematics concepts.

Effective teaching of mathematics includes use of appropriate teaching and learning tools such as Information and Communication Technologies (ICTs). Incidentally, narratives suggested that MTE are expected to have substantial knowledge of how to guide student teachers to adapt teaching and learning tools for effective teaching of mathematics. Likewise, teacher educators require understanding of the varied frameworks of teacher knowledge, instructional theories, and theories of learning mathematics. Teacher knowledge frameworks could facilitate teacher educators to be abreast with the unique knowledge which teachers of mathematics are required to acquire. Whereas learning theories may inform teacher educators regarding how pupils learn mathematics, instructional theories could equip them with capacity to facilitate student teachers' ability to enhance pupils' achievement in mathematics. This is considering research evidence confirming that integration of principles of instructional theories when teaching mathematics topics does bear a positive impact on learners' achievement in mathematics (Malambo et al., 2023).

Admittedly, MTEK is still an under researched area (Beswick & Goos, 2018). However, there is recognition that mathematics teacher educators do influence the development of teachers of mathematics (Kenney et al., 2023). In this context, narratives from the current study suggested that MTE should generally be acquainted with what goes on in school mathematics classrooms. Moreover, research has shown that despite student teachers of mathematics having studied advanced mathematics in university, they do demonstrate misconceptions of concepts in school mathematics topics (Malambo, 2020, 2021). This evidence rationalizes the view that MTE should not only be familiar with mathematics challenges experienced by student teachers but should equally have the capacity to resolve the challenges. Such evidence also augments Kenney et al.'s (2023) perspective that MTE have a responsibility to, for example, particularly support mathematics teachers' development of effective research-based formative assessment practices. Overall, MTE require knowledge that would enable them to train teachers for effective teaching of mathematics topics.

It should be emphasized that while teachers of mathematics have opportunities to train for the teaching responsibility, MTE do not have deliberately designed programs to prepare them for the task of training teachers. Some mathematics teacher educators (like the two participants in the present study) have taught mathematics at school level before becoming teachers of mathematics teachers. Nevertheless, Murray and Male (2005) contend that the transition from a teacher of mathematics to being a mathematics teacher educator is not an easy one. The mathematics teacher educators involved in the current study explained that what qualified them to train teachers of mathematics was knowledge acquired through pursuit of postgraduate degrees. This disclosure seems to support the idea of having intentionally designed postgraduate programs in universities for prospective MTE. The notion of such unique postgraduate programs sounds reasonable in view of research findings which show that majority of the doctoral graduates in mathematics education related fields favor to be employed in academia (Glasgow, 2000; Kordestani et al., 2019). MTE concurrently recognized the role that mentorship, professional associations, workshops, and seminars played in enabling them to progressively acquire the necessary MTEK. This revelation partially corroborates Anekstein and Vereen's (2018) view concerning the significance of research mentorship. It equally seems like a step in addressing Beswick and Goos's (2018) proposal to consider ways in which MTEs become knowledgeable.

Conclusion

This article's focus was characterized by conceptions of the MTE regarding what ought to compose the MTEK. Prominent understandings of MTE suggested that MTEK must incorporate knowledge of research in mathematics teacher education and teaching, mathematics teacher knowledge, and various MTE professional development activities. Furthermore, postgraduate studies, on the job practice, mentorship, and engagement in teacher education associations were considered as sources of MTEK. An additional finding that emerged from the narratives is that the undertaking to exhaustively categorize that knowledge is complex. The scarcity of research concerning MTEK (Beswick & Goos, 2018; Oates et al., 2021) was also affirmed. Scarcity in that sense suggests the prominence of a tacit belief that once MTE attain postgraduate qualifications, they inevitably acquire knowledge which enables them to universally train mathematics teachers. That salient belief has persisted despite the reality that contents of postgraduate programs are varied across countries and teacher training institutions (Tatto & Senk, 2011).

Recommendations

Research studies and in different contexts are inevitable to inform us of other categories of MTEK. All in all, the findings reported in the current article call for mathematics teacher education researchers to continue investigating how MTE should acquire MTEK.

Limitations

Findings reported in this article are based on a case study involving only two mathematics teacher educators practicing at a single university and who agreed to participate in the study.

Ethics Statements

Sampled mathematics teacher educators gave informed consent to participate in the study.

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Conflict of Interest

There is no conflict of interest

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References

Anekstein, A. M., & Vereen, L. G. (2018). Research Mentorship: Implications for the Preparation of Doctoral Students. *The Journal of Counselor Preparation and Supervision*, *11*(2), Article 6. https://core.ac.uk/download/pdf/234958357.pdf

Australian Institute for Teaching and School Leadership. (2015). Accreditation of initial teacher education programs in Australia: Standards and procedures. AITSL. https://bit.ly/3WQ78Bh

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching what makes it special? *Journal of Teacher Education*, *59*(5), 389-407. https://doi.org/10.1177/0022487108324554

Belbase, S. (2019). A Comparative Study of Mathematics Education in the United States of America and Nepal. *Mathematics Education Forum Chitwan*, *4*(4), 1-20. https://doi.org/10.3126/mefc.v4i4.26355

Beswick, K., & Chapman, O. (2012). Discussion group 12: Mathematics teacher educators' knowledge for teaching. Conducted at the 12th International Congress on Mathematics Education held in Seoul, South Korea.

Beswick, K., & Goos, M. (2018). Mathematics teacher educator knowledge: What do we know and where to from here? *Journal of Mathematics Teacher Education*, *21*, 417–427. https://doi.org/10.1007/s10857-018-9416-4

Chapman, O. (2008). Mathematics teacher educators' learning from research on their instructional practices. In B. Jaworski & T. Wood (Eds.), *International handbook of mathematics teacher education*: *Vol. 4: The Mathematics Teacher Educator as a Developing Professional*, (pp. 110–129). Sense Publishers.

Dede, Y., & Soybaş, D. (2011). Preservice mathematics teachers' experiences about function and equation concepts. *Eurasia Journal of Mathematics, Science & Technology Education*, *7*(2), 89-102. https://doi.org/10.12973/ejmste/75183

Dreher, A., Lindmeier, A., Heinze, A., & Niemand, C. (2018). What Kind of Content Knowledge do Secondary Mathematics Teachers Need? A Conceptualization Taking into Account Academic and School Mathematics. *Journal für Mathematik-Didaktik*, *39*, 319-341. https://doi.org/10.1007/s13138-018-0127-2

Evangelidou, A., Spyrou, P., Elia, I., & Gagatsis, A. (2004). *University students' conceptions of function*. In *The Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* (Vol 2, pp 351–358). Bergen University College.

Glasgow, R. (2000). *An investigation of recent graduates of doctoral programs in mathematics education* (Publication No. 9999287) [Doctoral dissertation, University of Missouri-Columbia]. ProQuest Dissertations and Theses Global. Unpublished doctoral dissertation.

Kenney, R. H., Lolkus, M., & Maeda, Y. (2023). Formative assessment in secondary mathematics: Moving theory to recommendations for evidence-based practice. *Mathematics Teacher Educator*, *11*(2), 76-92. https://doi.org/10.5951/MTE.2022.0019

Kordestani-Moghaddam, A., Esmaillzadeh, A., & Azadbakht, L. (2019). Postgraduate Research Mentorship Program: An approach to improve the quality of postgraduate research supervision and mentorship in Iranian students. *Journal of Education and Health Promotion*, *8*(1). Article 109. https://bit.ly/42ow6IO

Loughran, J. (2014). Professionally developing as a teacher educator. *Journal of Teacher Education*, *65*(4), 271–283. https://doi.org/10.1177/0022487114533386

Malambo, P. (2020). Pre-service mathematics teachers' nature of understanding of the tangent function. *JRAMathEdu (Journal of Research and Advances in Mathematics Education),* *5*(2), 105-118. https://doi.org/10.23917/jramathedu.v5i2.10638

Malambo, P. (2021). Implicit misconceptions in prospective mathematics teachers' reasoning about trigonometric concepts. *Contemporary Mathematics and Science Education*, *2*(2), Article ep21011. https://doi.org/10.30935/conmaths/11054

Malambo, P., Kazika, G. M., & Phiri, P. A. (2023). Impact of the activity and reality principles on learners' achievement regarding systems of linear equations. *Journal of Mathematics and Science Teacher*, *3*(1), Article em022. https://doi.org/10.29333/mathsciteacher/12609

Malambo, P., van Putten, S., Botha, H., & Stols, G. (2018). Mathematics student teachers' understanding of trigonometry for secondary schools. *11th Annual International Conference of Education, Research and Innovation*. https://doi.org/10.21125/iceri.2018.0748

Malambo, P., van Putten, S., Botha, H., & Stols, G. (2019). Dysfunctional functions: The case of Zambian mathematics education students. *Eurasia Journal of Mathematics, Science and Technology Education*, *15*(1), Article em1651. https://doi.org/10.29333/ejmste/99510

Merriam, S. B. (2009). *Qualitative research a guide to design and implementation* (2nd ed.). Jossey-Bass.

Murray, E., Baldinger, E., Wasserman, N., Broderick, S., & White, D. (2017). Connecting advanced and secondary mathematics. *IUMPST: The Journal*, *1,* 1-10.

Murray, J., & Male, T. (2005). Becoming a teacher educator: Evidence from the field. *Teaching and Teacher Education*, *21*(2), 125–142. https://bit.ly/43j49DR

National Council for the Accreditation of Teacher Education. (2008). *Professional standards for the accreditation of teacher preparation institutions*. NCATE. https://bit.ly/3oHFJop

Nieuwenhuis, J. (2014a). Qualitative research designs and data gathering techniques. In K. Maree (Ed.), *First Steps in Research* (14th ed., pp. 69-97). Van Schaik.

Nieuwenhuis, J. (2014b). Introducing qualitative research. In K. Maree (Ed.), *First Steps in Research* (14th ed., pp. 46-68). Van Schaik.

Nieuwenhuis, J. (2014c). Analysing qualitative data. In K. Maree (Ed.), *First Steps in Research* (14th ed., pp. 98-122). Van Schaik.

Oates, G., Muir, T., Murphy, C., Reaburn, R., & Maher, N. (2021). What Influences Mathematics Teacher Educators' Decisions in Course Design: Activity Theory and Professional Capital as an Investigative Approach. In M. Goos, K. Beswick (Eds.), *The Learning and Development of Mathematics Teacher Educators, Research in Mathematics Education* (pp. 345–366). Springer. https://doi.org/10.1007/978-3-030-62408-8_18

Peng, A., & Luo, Z. (2009). A Framework for Examining Mathematics Teacher Knowledge as Used in Error Analysis, *For the Learning of Mathematics, 29*(3), 22-25. https://www.jstor.org/stable/25594562

Schuck, S. (2002). Using self-study to challenge my teaching practices in mathematics education. *Reflective Practice: International and Multidisciplinary Perspectives*, *3*(3), 327–337. https://doi.org/10.1080/1462394022000034569

Shulman, L. S. (1986). Those Who Understand: Knowledge Growth in Teaching. *Educational Researcher, 15*(2), 4-14. https://doi.org/10.2307/1175860

Tatto, M., & Senk, S. (2011). The Mathematics Education of Future Primary and Secondary Teachers: Methods and Findings from the Teacher Education and Development Study in Mathematics. *Journal of Teacher Education*, *62*(2), 121-137. https://doi.org/10.1177/0022487110391807

Zambia Ministry of Education. (2013). *Zambia education curriculum framework*. UNESCO UNESDOC. https://unesdoc.unesco.org/ark:/48223/pf0000230893

Zaskis, R., & Leikin, R. (2010). Advanced Mathematical Knowledge in Teaching Practice: Perceptions of Secondary Mathematics Teachers. *Mathematical Thinking and Learning*, *12*(4), 263-281, https://doi.org/10.1080/10986061003786349