Research Article https://doi.org/10.12973/ejmse.5.1.51

European Journal of Mathematics and Science Education
Volume 5, Issue 1, 51 - 65.
ISSN: 2694-2003
http://www.ejmse.com/

Understanding Problem-Based Learning and its Application in Learning Mathematics Concepts Among Pre-Service Teachers

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Received: August 9, 2023 ▪ Revised: December 23, 2023 ▪ Accepted: March 11, 2024

Abstract: Learning to teach mathematics has become crucial since its application in real life cannot go unmentioned. The desire of mathematics education researchers to make mathematics concepts easier for pre-service teachers to easily understand has attracted attention. This has become indispensable since after college, pre-service teachers are deployed from K-12 to assist learners in understanding mathematics concepts. The study aimed to ascertain how improvement in the learning of mathematics concepts using the Problem-based learning (PBL) approach could be understood and/or explained among pre-service teachers. This was viewed in two folds: how improvement in learning outcomes using the PBL approach could be explained; and how pre-service teachers' disposition about the PBL could be explained/understood. Exploratory case study design involving qualitative and quantitative data was concurrently gathered and used. This involved the use of data collection instruments such as focus group discussion, pre-post-test scores, PBL observation protocol, and PBL disposition questionnaire. The study showed that the PBL method improved the learning of mathematics concepts among pre-service teachers. Pre-service teachers also showed a positive disposition (interest, belief, and attitude) toward the PBL intervention. The authors advocated for the conduct of a longitudinal study to understand the direction of change over time.

Keywords: *Mathematics education, problem-based learning, small-group activity.*

To cite this article: Boye, E. S., & Agyei, D. D. (2024). Understanding problem-based learning and its application in learning mathematics concepts among pre-service teachers. *European Journal of Mathematics and Science Education*, *5*(1), 51-65. https://doi.org/10.12973/ejmse.5.1.51

Introduction

In these contemporary times, facilitating the understanding of mathematics concepts among learners from K-12 largely depends on the content knowledge and pedagogical skills of pre-service teachers. This has instigated the need for pre-service teachers to understand and adapt to instructional approaches that build team spirit and problem-solving skills among learners. The need for tutors in teacher training institutions to produce pre-service teachers with appreciable critical thinking and problem-solving skills has become indispensable. According to the Ghana Ministry of Education (2019), the current emphasis is for pre-service teachers to acquire instructional competence in preparation for their future careers. Thus, the new paradigm expects teacher trainees to construct their understanding, apply relevant skills and previous knowledge, and see the purpose in what they learn.

Some studies have revealed that most tutors from colleges of education in Ghana use the traditional method of instruction to teach mathematics concepts (Enu et al., 2015). Further studies have uncovered that basic school mathematics teachers in Ghana are noted for the use of traditional methods in mathematics instruction (Fletcher, 2003; Ottevanger et al., 2006). Interestingly, the approach continues to be used in contemporary times due to the comparable economic benefits they bring to learning institutions. According to Ossei-Anto et al. (2013), an examination conducted at the University of Cape Coast by the Institute of Education revealed that over 50% of the examinees failed in Mathematics and Science. This disturbing phenomenon has been attributed to numerous factors including poor pedagogical skills and focusing more on chalk-and-talk instead of adopting the constructivist approach. In Ghana, Colleges of Education are tertiary institutions established to train professional teachers for subsequent deployment at elementary schools. Teacher education in Ghana

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has experienced numerous modifications in the past decades. These reforms are due to policy changes geared toward grooming teachers to address the country's educational needs.

Adunola (2011) posited that the use of ineffective teaching methods by teachers to impart content to learners is the main reason for poor academic performance. An extensive study into the efficacy of teaching methods reveals that the quality of instruction is assessed based on the results obtained from the learners. Mereku (2008) in a study revealed that Ghana, like most developing countries, employs the traditional method of teaching mathematics in our classrooms. Mereku further indicated that the traditional method of teaching relies mainly on textbooks thereby promoting memorisation at the expense of the practicality of the strategy. It is in light of this that problem-based learning (PBL), a modern and learner-centred method of teaching was explored in this study.

Finkle and Torp (1995) viewed PBL as a curriculum growth and a system of instruction that concurrently improves problem-solving and disciplinary knowledge bases and skills. This could be achieved by making learners more active than passive when confronted with authentic tasks that mirror real-world problems. Hmelo-Silver (2004) argued that the unambiguous aim of PBL is to advance learners' capabilities in their needed skills for work or social life. Such skills may include problem-solving and assessment, communication, and teamwork. PBL was first implemented at McMaster University, Ontario, Canada in medical education (Barrows, 1996).

The PBL concept has become popular in the teacher education literature in recent times. Levin (2001) argued for the integration of the PBL strategy in the teacher education curriculum to make learning more engaging and relevant for pre-service teachers. Similarly, Dean (1999) considered PBL as a necessary tool to help teachers impart the needed skills and knowledge to pre-service teachers as professionals. Dean asserted that using an instructional method like a PBL that encapsulates the tenets of constructivism could facilitate lesson delivery. Earlier studies into the implementation of PBL into teacher education programmes have shown that the teaching method facilitates teacher trainees' ability to relate theory to practice. Padmavathy and Mareesh (2013) asserted that PBL has a positive effect on teaching mathematics and could increase learners' comprehension and aptitude to apply the concepts in everyday life. Unfortunately, much cannot be said about PBL implementation in mathematics education in today's classrooms and colleges of education in Ghana. The purpose of the study is to examine pre-serve teachers' understanding of PBL and their application in learning mathematics concepts. The study is focused on ascertaining how improvement in pre-service teachers' learning of mathematics concepts using the PBL approach could be understood and/or explained**.** This was viewed in two folds: how improvement in learning outcomes could be explained; and how pre-service teachers' disposition about the PBL could be explained/understood.

Theoretical Framework

The theory underpinning the study is Lev Vygotsky's social constructivism which focuses on learners' thoughts. The theory asserts that for learners to develop cognitively, they must be guided and mediated by the way they socially interact. Vygotsky's theory posits that learning is a social process hence learners must interact socially to facilitates and develop their cognition and interaction. Vygotsky (1978) argued that learners acquire basic knowledge when they compare, share, and debate among themselves and their facilitators in groups. By this, learners could refine the meaning of concepts and help others to mutually construct knowledge. The theory reflects and highlights the support facilitators provide learners in acquiring knowledge and skills that are more complex for learners to understand individually. Vygotsky's theory, which is a subset of the theory of constructivism, projects student-centredness and cooperative teaching and learning styles. This enables learners to tackle authentic tasks with the support of knowledgeable peers (Dagar & Yadav, 2016).

In a PBL mathematics classroom, Lev Vygotsky's theory could promote effective instruction by encouraging communication and interaction between a small group of learners and a facilitator. Peer cooperation and small group activities characterise the theory in the PBL mathematics classroom. The assistance and guidance given to the learners by their peers and facilitator make the learning of mathematics concepts comprehensible and engaging. Lev Vygotsky (1978) initiated the Zone of Proximal Development (ZPD) concept which describes tasks that learners cannot master alone but can learn with the support of skilled peers, adults, or facilitators. This socio-cultural theory together with the ZPD idea hypothesises that a PBL lesson amalgamates cooperation, small group activity, and interaction among students and instructors. Others include learner-centredness, and communication skills and these could stimulate effective teaching and learning of mathematics concepts among teacher trainees. The theory further suggests that facilitators must design exercises that can motivate and inspire less skilled learners to develop with the support of their knowledgeable peers and facilitators. This requires varying the level of support and the difficulty level of the exercise until that time when the learner becomes confident to solve problems independently.

The theory implies that interactive teaching methods that put learners in active groups should be employed by teachers to facilitate learning. It further implies that learners cooperate in teams and provide platforms for students to seek clarification and communicate feedback on lessons learnt. The authors argue that teacher trainees ought to learn in small groups to help them derive support from their peers to enable them acquire appreciable skills and knowledge on PBL.

This will help them to prepare and use a PBL lesson plan that reflect the features of the PBL strategy for mathematics instruction.

Conceptual Framework

The conceptualization of the PBL approach rests greatly in the appearance, practicality and characteristics of the pedagogy. The following characteristics of PBL pioneered by Barrows (1996) informed the conceptual framework:

(1) The learner is positioned at the centre of learning. Thus, the learner is the focus of the instruction in the PBL process and they are engaged in self-directed learning.

(2) Learners are put in small groups. The PBL process involves putting learners in small groups to brainstorm and discover new knowledge and skills for themselves.

(3) The instructor plays the role of a facilitator or a guide. Barrows and Tamblyn (1980) asserted that the instructor is to be proficient in posing precise unrestricted questions that are valuable in guiding learners through the cognitive process. It defines the capacity to direct the pace and to measure the learning process.

(4) The learners are presented with authentic problems from the onset of the learning. This helps to find gaps in knowledge in a PBL classroom.

(5) Achieving the goal and mastering the subject depend on the nature of the problem under consideration.

(6) The learners eventually gain knowledge when engaged in self-directed learning.

The authors are of the view that the characteristics proclaimed by Barrows (1996) in addition to his previous related studies (Barrows, 1992; Barrows & Tamblyn, 1980) reflect constructivism. This can be associated with the views of Vygotsky (1978) and consequently the conceptual framework underpinning the study.

The conceptual framework demonstrates the connection between the dependent and independent variables. The PBL strategy has been conceptualised to include learner-centredness, learners working in small groups, and cooperation among learners. Others include interaction between learners and facilitators, and communication skills (see Figure 1). The authors envisage that if mathematics teachers plan and implement PBL lessons guided by these six features, learning outcomes will improve. The framework in Figure 1 depicts the relationship between the features of the PBL strategy (independent variables) and improved learning outcomes (dependent variables). The succeeding paragraphs after Fig 1 review the features of the PBL.

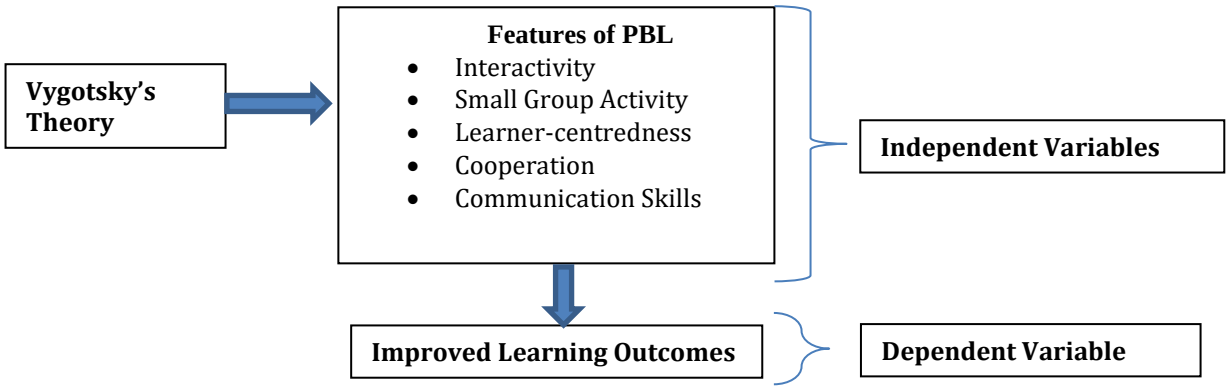

Figure 1. Proposed Conceptual Framework

Learner-Centredness

Learner-centred method of instruction has been widely proclaimed to correlate positively with learner achievement even though the method has not been universally defined (Mtika & Gates, 2010). But the common denominator is that learner-centred education stresses social interaction and mirrors individual student's active learning and advancement (Lattimer, 2015; Schweisfurth, 2013). Gonzalez (2019) and Passman (2000) suggested that the learner-centred teaching approach can be beneficial in several ways. It can increase learner participation, boost learner confidence, nurture intellectual growth, and direct learning tasks to learners. Muzumara (2011) argued that learner-centred activities encourage learners to engage in certain practical activities that enable learners to solve problems and answer formulated problems. These activities may take the form of projects, brainstorming, textbook study and the like which are good for solving mathematics problems. Weimer (2002) prescribed five practice areas that require regular changes to create learner-centredness during instruction. These include (i) the function of content (to build a cogent knowledge base and be able to apply independently); (ii) the processes and purposes of assessment (to provide students with constructive feedback to aid improvement); (iii) the balance of power (instructor gives students some control over expressing perspectives and their methods of learning); (iv) the role of the instructor (to assist students to learn by creating an

enabling environment); and (v) the responsibility for learning (instructor assists students to take responsibility for their own learning).

Students learn mathematics when they can adapt their previous learning to new information through their own meaning-making (Fosnot, 2005; Stigler & Hiebert, 2009). To offer students these opportunities to construct knowledge, purposeful tasks must be assigned (Fosnot, 2005; Stigler & Hiebert, 2009). In the case of the PBL, because students are to exhibit their comprehension at every stage of the lesson to inform progress, formative assessments of various forms were regularly used to measure improvement in learning. In this study, lessons were carefully designed to ensure that the learners engaged in brainstorming activities that made them active while the instructor served as a facilitator.

Interaction Among Learners and Facilitators

Interactivity helped to conceptualise the PBL process in this study. Reutova (2012) opined that to inspire interaction among learners about the advancement of learner activity, interactive teaching and learning methods remain key while acknowledging the role of the instructor. Kutbiddinova et al. (2016) argued that the learner mostly engages in dialogue with the instructor and other learners of the pedagogical process. When learners perform tasks in groups and receive regular feedback from their instructor, the interaction among them is realised. Schweisfurth (2013) asserted that learning is posited within a cultural norm and based on interactions between learners and the instructor. Interactive teaching fundamentally revolves around the learners, the instructor, and the environment. These three factors continuously connect to facilitate interaction among them to influence each other. Students while learning convey information to the instructor through questions, attitudes, gestures, or behaviours. The instructor responds to the inquiries made by the learners and this helps to motivate the learners and bring about appreciable changes in instructions. By this, the instructor responds to the actions of the learners. This justifies the interaction between the learners and the instructor.

The environment remains a factor that constantly affects the instructor and the learners and creates either favourable or unfavourable conditions for the learners (Minh, 2018). Undoubtedly, an unfavourable condition may create discomfort for the learners and distract their attention. Thus, learning outcomes could be evaluated through the interaction between the environment, students, and the instructor. The instructor's role in interactive learning is aimed at achieving the goals of learners in the teaching enterprise. The instructor makes an interactive lesson plan and interactive exercises and tasks to elicit desired teaching and learning outcomes. Interactive teaching methods ensure the full participation of students in the learning process, which is a major source of learning (Minh, 2018). In this study, the instructor acted as a facilitator (Savery, 2019). The PBL Mathematics lessons were carefully designed to be interactive and inspired pre-service teachers to express their opinions on the topics taught. Learners with varied learning abilities could also relate with their contemporaries. This approach encouraged learner interaction with their facilitator, thus growing their commitment to the study materials.

Small Group Activity

Small group activity has been identified as a strategy employed in instruction to create means for students to share ideas and find solutions to a problem. According to Meo (2013), putting learners in small groups and presenting them with a problem is an effective means of imparting knowledge to learners. This could lead to expected learning outcomes. Gaudet et al. (2010) asserted that several studies have revealed that small-group learning could increase the retention and knowledge of learners. This enhances their performance and attitude toward learning. Putting learners in small groups can promote student-centred curricula and develop their social skills aside from encouraging them to collaborate as members of a social cohort. This will help them learn, construct and share their newly acquired information (Dolmans et al., 2001; Eva, 2002; Holen, 2000). McClean et al. (2006) conducted a study that focused on first-year medical students' perceptions of small group involvement in PBL at the start of their PBL lessons. According to the study, a lot of students argued that their experiences had led to their cognitive and personal development. This development extends from becoming tolerant and confident to becoming self-assured.

The facilitator coaches the group by stimulating discussion among group members and encouraging students to articulate their views. Thus, the facilitator is encouraged to promote debate and critical thinking, ask clear and open-ended questions, and give constructive feedback (Edmunds & Brown, 2010). Despite this, some scholars have challenged the efficacy of PBL especially among dysfunctional groups (Norman, 2001) where intolerance of individual differences may adversely affect learning (Hitchcock & Anderson, 1997). In a PBL mathematics classroom, facilitators must observe and direct activities within and among groups to enable the small groups to operate effectively. In this study, the PBL mathematics class enrolled 32 first-year pre-service teachers. The potential teachers were put into small groups of between 4-8 and taught some mathematics lessons using a carefully designed PBL lesson plan.

Cooperation Among Learners

Cooperation has also been incorporated as a feature of the proposed PBL strategy in this study. Srinivas (2011) argued that learners can exchange ideas with peers, share different views, justify thoughts, investigate other frameworks, and actively engage in a cooperative setting. Consequently, Laal (2013) asserted that in a cooperative classroom setting, members of a study group should be answerable for the success or otherwise of the assigned tasks as well as the mastery of all the material to be learnt.

Cooperative learning promotes thinking and permits students to communicate their thinking. Through classroom discourse, students will develop a stronger sense of mathematics (Lappan et al., 2014; Smith & Stein, 2011). Cooperative learning aims at accomplishing a common goal and this encourages learning in small groups to help one another to accomplish the desired goal. Thus, the responsibility lies on each student to share views and work as a team to solve the mathematical problem at hand.

Evidence from previous studies has proved that cooperative learning can improve students' performances. This is in addition to growing the interaction and social skills among learners and instructors (Zakaria & Iksan, 2007). More so, cooperative learning encourages critical thinking, promotes mutual respect and boosts confidence among students (Artzt & Newman, 1997; Gonzalez, 2019; Liu & Pásztor, 2022; Slavin, 1995). Long (2010) posited that cooperative learning stimulates students' cognitive abilities, thereby changing the conventional method of teaching. Long also stated that cooperative learning challenges learners to generate and voice their opinions on relevant topics learnt. The method is an effective way to produce active learners.

Furthermore, promoting cooperation among students has evolved in mathematics education literature. Sin (2006) in cooperative learning settings involving 60 form-one students used the Students Team Achievement Division (STAD) model to teach Mathematics. This revealed that the students performed better when they were introduced to cooperative learning as compared to the traditional method. Tarim and Akdeniz (2008) conducted a similar study which revealed the effects of cooperative learning on Turkish elementary students' mathematics achievement and attitude towards mathematics. The study used Team Assisted Individualisation (TAI) and Students Team Achievement Division (STAD) methods. The findings revealed performance improvement. Again, Zakaria et al. (2010) in a study involving two form-one classes in Miri, Sarawak discovered that the cooperative learning method improved students' mathematics performance.

The authors of this paper argue that cooperative learning encourages individual and group collaboration to achieve shared teaching-learning objectives. The PBL approach emphasises group activity and benefits not only the academic but also the learner's social personality. In a classroom, students have the opportunity to develop cross-ethnic friendships, engage in fruitful interaction, enhance their communication and problem-solving skills, and foster critical thinking skills.

Communication Skills

Communication skills are crucial in learning mathematics concepts. This requires attention since the skills could help understand and organise learners' mathematical thinking (Saragih, 2007). Mathematical communication skills have largely been defined as the ability of students to comprehend mathematical definitions, notations or symbols correctly and be able to express ideas through written, oral, and/or graphical depictions. According to Hulukati (2005), finding an answer to mathematics problems requires effective communication skills. This will prepare learners to articulate opinions and answers properly and explain mathematics concepts and problems. Baroody and Coslick (1993) opined that students can develop a better grasp of mathematics concepts by paying attention to what their peers share in a group setting. Reflecting and explaining how students think about mathematical ideas and relationships require language and symbols in mathematics to develop and communicate the mathematics concepts (Sumarmo, 2000).

Demonstrable good communication skills are often expected among students who participate in PBL. The students often gather understanding and application of their knowledge in practice better and the complications of other issues involved in the practice of communication. In the PBL approach, effective planning, organisation and monitoring could significantly promote communication and collaborative teamwork skills.

Various forms of communication are considered to explore and deepen students' understanding of mathematical concepts and create a link between other fields of knowledge. These mathematical ideas in any form can help develop students' mathematical abilities. Students are therefore encouraged to talk more, elucidate concepts, and relate ideas in real situations to construct meaning (Van de Walle et al., 2010). Thus, giving mathematics problems related to real-life situations is very necessary to deepen students' understanding of concepts and to develop their communication skills. Furthermore, employing peer learning techniques and game-learning activities can also be used by facilitators to develop mathematical communication skills. Qohar and Sumarmo (2013) argued that students will be able to express coherent mathematical ideas, find friends, and develop spoken and written language through good mathematical communication skills. To wit, encouraging group members in a PBL mathematics class to express ideas on a mathematics concept was evident in this study. This did not only enhance student learning but provided learners with appreciable skills.

The Problem-Based Learning Intervention

The data collection covered one month following the intervention of the PBL strategy. The authors designed and taught eight mathematics lessons (see Table 1) using the PBL approach. At the beginning of the PBL lessons, a meticulously crafted lesson plan was used to guide the delivery of the lessons to the students. The lesson plan was designed to reflect the features of PBL as projected in the context of the study. The enactment took into consideration the proposed conceptual framework (see Figure 1) and the PBL characteristics pioneered by Barrows (1996). Barrows also proposed some steps in implementing the PBL strategy which the authors adapted and used in the conduct of the study. The steps include:

a. The pre-service teachers while in groups were presented with an authentic mathematics problem to solve.

b. The instructor attempted to elicit the understanding of the pre-service teachers by identifying their knowledge gap about the problem and what was expected of them. The instructor defined the problem and provided guidance where necessary.

c. The pre-service teachers were guided by the instructor to demonstrate cooperation, interactivity, learner-centredness, and effective communication skills while sharing information in groups and with the instructor. The instructor facilitated the group activity to ensure that the students were able to work together effectively and achieve their learning objectives.

d. The teacher trainees generated and presented possible solutions to the instructor. The best approach to the problem was highlighted by the instructor through feedback. This was followed by a debriefing on the problem by the instructor.

The authors of this paper at the onset administered a pre-test to the students for each lesson that was taught. The outcome of the pre-test helped to assess the previous knowledge of the pre-service teachers. The questions focused on some selected senior high school topics in core mathematics and covered lower-order thinking (knowledge, comprehension, and application) in line with Bloom's (1956) taxonomy. After, the students were put into groups with group members ranging from 4 to 8 in a semi-circle or such that the groups could see the instructor. Post-tests were conducted at the end of each lesson. These tests formed part of the data obtained in answering the research questions. The instructor collected data before, during, and after each PBL lesson. After the completion of all eight lessons, the instructor distributed a questionnaire to the pre-service teachers to gather feedback on their opinions and disposition. Furthermore, the instructor together with the participating college tutors made some observations during and after the PBL process. The observations were recorded in the researcher logbook coupled with some video recordings. More so, focus group discussions involving eight students were held.

Table 1. Lessons Taught Using Problem-Based Learning Strategy

Lesson	Sub-strand	Class/Level	Duration (minutes)
L1	Ratio and Rates	Level 100	120
L2	Percentages 1	Level 100	90
L3	Percentages 2	Level 100	90
L4	Discount and Commission	Level 100	120
L5	Simple Interest, Profit & Loss	Level 100	120
L6	VAT, NHIS & Insurance	Level 100	120
L7	Relations and Functions	Level 100	120
L8	Composition of Functions	Level 100	120

Research Questions

The study sought to ascertain improvement in pre-service teachers learning of mathematics concepts using the PBL approach. This was framed into two research questions as follows:

- How can improvements in learning outcomes using the PBL approach be explained/understood?
- How can pre-service teachers' disposition about the PBL approach be explained/understood?

Methodology

Research Design

An exploratory case study design was employed because it helps to investigate a new phenomenon using many sources of data (Yin, 2003). The literature consulted could not explicitly indicate that the PBL strategy is used in Ghana for teaching pre-service teachers mathematics. This makes the method somewhat new in mathematics instruction at the teacher training colleges in Ghana. The study design was predominantly qualitative, but quantitative evidence was used to support the findings for generalization. Thus, the authors collected and used both qualitative and quantitative data concurrently.

Study Area, Population, and Sample Size

The study was conducted at Akrokerri College of Education. The College is one of the 48 teacher training colleges in Ghana responsible for teacher education. It is situated in the Ashanti region of Ghana and has several teacher education programmes that focus on lower primary, upper primary, and junior high school education. Akrokerri College is noted for excelling in mathematics and science programmes, thereby making the pre-service teachers competent to handle basic school mathematics after college. The population of the students in the school was 1, 500 with 40 students making the average class size. The College also had nine first-year classes. The selected class (intact class) had 32 students and this intact class was the case study group. The group helped to facilitate a comprehensive enquiry into the efficacy of the PBL approach. The class comprised 19 females and 13 males.

Sampling Techniques

In this study, the authors used purposive sampling technique to select Akrokerri College of Education and two mathematics tutors as participant observers. Purposive sampling was used because the authors, based on their judgements, noticed that the participants possessed certain characteristics that could help address the research objectives/questions. Such characteristics include the fact that the mathematics tutors are beneficiaries of the Japan International Cooperation Agency (JICA) sponsored Science, Technology and Mathematics project which allowed them to bring their pedagogical experience to bear. One out of the nine first-year classes was selected using a simple random sampling technique, and this class formed the intact class that made up the sample size. This was because the authors decided to give all nine classes an equal chance of participation.

Data Collection Instruments

Qualitative and quantitative data were collected from the pre-service teachers and the college tutors who were directly involved in the PBL intervention. Data collection instruments such as a structured questionnaire for pre-service teachers; focus group discussions for pre-service teachers; observation protocol for tutors; and pre-test and post-test scores for pre-service teachers were employed accordingly in the study.

Pre-Service Teachers' Problem-Based Learning Disposition Questionnaire

The questionnaire sought to find out pre-service teachers' attitudes, beliefs, and interests in the PBL lessons taught. In this study, *attitude* refers to the way students' participants think and feel about the PBL intervention; *belief* refers to teacher trainees' opinion about the truthfulness of the PBL approach, and; *interest* also refers to the feeling that the pre-service teachers desire to know or learn more about the PBL approach. The questionnaire comprised 21 items grouped under three subscales: attitude (5 items), belief (9 items), and interest (7 items) and evaluated using a five-point Likert scale (strongly disagree = 1, disagree =2, not sure =3, agree =4, and strongly agree =5). The students chose one of the responses ranging from 1 to 5. A score below 3 showed a negative opinion while a score of 3 or above showed a positive favourable opinion. Analysis of the data showed Cronbach's alpha of 0.51, 0.713, and 0.661 for attitude, belief, and interest respectively. This means that the reliability coefficient met the minimum criteria and followed the main recommendations of DeVellis (1991).

Focus Group Discussion

The authors moderated a group discussion comprising eight participating first-year students chosen using purposive sampling from the case study group. The discussions lasted for 90 minutes and the responses were recorded, transcribed, and analysed qualitatively. The focus group discussion helped to discover some challenges about the PBL approach which were not meant to condemn the instructional strategy but to give room for improvement in subsequent implementation. The outcome of the discussions is summarised in Table 3.

Problem-Based Learning Observation Protocol for College Instructors

The authors used observation protocol to gather qualitative evidence for the study. This requires a coherent account of the behaviours, events, and artefacts of social background (Marshall & Rossman, 1989). The observation protocol also

helps to learn what is important about the people in the social setting under study. In this study, two college tutors and the authors keenly observed the learners to determine their reaction toward the learning of the mathematics concepts presented to them using the PBL approach. The questions in the observation protocol/guide were responded to by the tutors and this helped to examine how the improvement in learning mathematics concepts using the PBL approach could be understood and/or explained.

Pre and Post Tests Scores

The authors used pre-post-tests which were essay questions and covered the topics provided by the college and taught by the authors. The authors conducted group exercises via an activity sheet for each lesson. Bloom's (1956) order of cognitive domain of learning informed the pre-post-tests. The pre-test covered the first three domains (Knowledge, Comprehension and Application) of learning whereas the post-tests focused on the last three domains (Analysis, Synthesis, and Evaluation) of learning. The mathematics topics for the Senior High School learners in Ghana were used as the yardstick for the pre-test and these permitted a speedy appraisal of basic knowledge. Conversely, the post-test measured the higher-order skills of the learners. It must be noted that the interpretation of the test scores contributed to achieving the objective of the study.

Data Analysis

The authors analysed the data collected via the pre-post-tests, pre-service teachers' disposition questionnaire, focus group discussion, and observation protocol. The test (i.e., pre and post) scores were scaled down to 5 for each of the tests. A score below 3 showed a negative opinion while a score of 3 or above showed a positive favourable opinion of the pre-service teachers' response. The authors used paired samples t-test to analyse the mean scores of the pre-post-tests. This research verified differences in pre-service teachers' achievements in the pre-post-tests. The pre-service teachers' responses to the designed disposition questionnaire were also analysed after the PBL lessons. All statistical tests were tested at a significant level of 0.05. This measurement fits all assumptions of analyzing techniques. Thus, the data within each group followed a normal distribution. The authors also used grounded theory analysis as postulated by Charmaz (2006) and content analysis (Morgan, 1988) to analyse data collected through focus group discussion after the PBL intervention. The authors were consistent with the research approach, ensured triangulation, and handled the qualitative data carefully to ensure the reliability of the study outcome. Additionally, Cronbach's alpha was used to estimate the reliability coefficient and this met the minimum criteria.

Findings/Results

The results are presented according to the two research questions stated to achieve the overarching purpose of the study.

Improvements in Learning Outcomes Using PBL Approach

In fulfilling the purpose of the study, the authors analysed the pre-post-test scores of the teacher trainees. Observations made by the authors, the discussions from the focus group, and the marked scripts of the trainees were also analysed to achieve the purpose.

Firstly, the authors used pre-post-test scores to ascertain how improvement in learning could be explained. The pre-post tests were conducted to ascertain whether there was an improvement in learning the mathematics concepts presented to the teacher trainees. The tests were conducted for all eight lessons that were taught. To ascertain the pre-service teachers' enhanced learning, the tests were conducted before and directly after the PBL lessons. Table 2 presents the mean scores of the pre-post-tests for the eight lessons taught.

Table 2. Results for Pre-Post-Test Average Mean Scores for Each Lesson (n=32)

Lesson	Pre-test	Post-test	Diff	Sub-topic
L1	1.955	4.123	2.168	Ratio and Rates
L2	2.014	2.937	0.923	Percentages 1
L3	1.663	2.789	1.126	Percentages 2
L4	1.367	3.910	2.543	Discount and Commission
L5	1.651	2.714	1.063	Simple Interest, Profit and Loss
L6	1.286	3.113	1.827	VAT, NHIL and Insurance
L7	1.108	2.560	1.452	Relations and Functions
L8	0.849	2.337	1.488	Composition of Function

Table 3. Overall Results of Paired Samples T-test for the Pre-Post-Test Scores

	M	SD	t	df	sig (2-tailed)	Effect size
Pair 1 Pre-post-test	1.57	.36	25.05	31	.000	4.36

From Table 3, there is a shred of strong evidence (t=25.05, P<0.05) that the teaching intervention improved learning outcomes. The estimated improvement is approximately 1.6 marks. Also, the effect size indicates that the relationship between the pre-tests and post-tests was significant.

Secondly, the authors, by way of measuring an improved learning outcome during the instructional period, employed some observation skills. For instance, they observed that the use of question-and-answer techniques helped to elicit teacher trainees' understanding of the mathematics concepts taught. In some instances, the trainees correctly answered challenging questions posed to them by the authors. Pre-service teachers' correct responses to the authors' questions confirmed learning had taken place, especially where it touched on areas that were mastered after the PBL intervention. This is also evident in the post-test scores in Table 2.

Thirdly, the use of focus group discussion also assisted in responding to the research questions of the study. The authors used this instrument to elicit responses on the improvement of learning outcomes using the PBL approach. The authors posed the following questions to the group and gathered opinions on pre-service teachers' learning outcomes:

Q1. Was the instructional method adequate to improve the understanding of mathematics concepts?

Q2. In general, do you think the teaching and learning method used has improved your knowledge, understanding, and level of confidence?

From the qualitative evidence gathered from the focus group discussion, it was revealed that there was an improvement in learning outcomes. Table 4 shows evidence of the extent to which this improvement can be explained.

Table 4. Responses from the Focus Group Discussions on the Improvement of Learning Outcomes

FGD	Respondent (R)	Improvement Dimension
Q1	R6	1. I think it was adequate because, after school, we were given questions to solve to understand the lessons better. Where we had challenges, we discussed them amongst ourselves, and we were able to arrive at the desired solution (***small group activity; cooperation; learner-centredness).***
	R2	2. It was enough because, before the start of each lesson, some questions were given to us to solve but we were not able to answer a lot. But after each lesson, we were able to answer a lot of the questions we were given (***cooperation; communication skills)***
Q2	R5	3. Yes because you encouraged us to talk and that helped to build our confidence ***(communication skills; interactivity)***
	R7	4. Yes because I do not normally talk during mathematics lessons but because of how you taught us I was able to answer questions. (***interactivity & communication skills***)
	R6	5. Some of us because of the way we were taught by our previous mathematics teachers, we thought mathematics was difficult but given this opportunity to work in groups and express our thoughts, our understanding has increased (***small group activity, interactivity & communication skills***)
	R1	6. Yes because I was not all that good in mathematics but because you put us into groups, I was able to seek understanding from my friends where I had difficulty understanding alone and so I can say that I am better now. (***Small group activity; learner-centredness; & cooperation***

Based on the responses received from the teacher trainees, it is clear that their responses reflected the constructs of PBL as indicated in Figure 1. To this end, it can be concluded that largely the intervention helped to improve learning outcomes.

Pre-service Teachers' Disposition of the Problem-based Learning Approach

The second research question which seeks to ascertain pre-service teachers' disposition toward the PBL intervention is addressed. Table 5 shows the descriptive statistics of pre-service teachers' disposition towards the PBL method.

Table 5. Descriptive Statistics of Pre-Service Teachers' Disposition of PBL (n =32)

Disposition Dimension	Mean (M)	Standard Deviation (SD)
Attitude	4.42	.596
Belief	4.41	.562
Interest	4.31	.594

Source: Field Data (2021) SA = 5, A = 4, NS = 3, D = 2, SD = 1

The pre-service teachers' disposition: Attitude (M = 4.42, SD = 0.596), Belief (M = 4.41, SD = 0.562), and Interest (M = 4.31, SD = 0.594) was positive. Thus, the students claimed the intervention helped them to learn the mathematics concepts they were introduced to.

In conclusion, the results displayed in line with the research questions suggest that there was an improvement in learning. Thus, the PBL intervention was effective and justified earlier studies conducted by some scholars (Balendran & John, 2017; Mughal & Shaikh, 2018; Yadav et al., 2018). This is because the students enhanced their learning of mathematics concepts which was evident in their improved learning outcomes and the students' positive disposition toward the PBL intervention.

Discussion

The study focused on how improvement in pre-service teachers' learning of mathematics concepts using the PBL approach could be understood and/or explained. This was viewed in two folds: (1) how improvements in learning outcomes using the PBL approach could be explained or understood; and (2) how pre-service teachers' disposition toward the PBL approach could be explained. In addressing the first part of this research purpose, the authors depended on data gathered via observation protocol, focus group discussion, and pre-post-test scores. Conversely, the pre-service teachers' disposition questionnaire was also used to conclude the second part of the research purpose.

In analyzing the pre-post-test scores, the authors used paired samples t-test. Table 3 shows strong evidence (t=25.05, $P<0.05$) that the teaching intervention improved learning outcomes. The estimated improvement was approximately 1.5 marks. Also, the effect size, d was approximately 4.43. It is noteworthy that the *p*-value and the effect size indicate statistical significance. The effect size also measures the strength of the relationship between two variables (pre-test and post-test scores). Cohen et al. (2017) proposed that effect size (*d*) near 0.2 is a small effect size, *d* near 0.5 is a medium effect size, and *d* near or greater than 0.8 is a large effect size. The study revealed an effect size of approximately 4.43. This could imply a huge effect size. Thus, the means of the pre-test and post-test differ by 4.4 standard deviations, thereby, making the impact of the PBL strategy very significant in promoting the learning of mathematics concepts among the teacher trainees. The improved learning outcomes as shown from the pre-post-test scores in Table 2 could be attributed to the PBL intervention which saw the involvement of the pre-service teachers working in groups. This helped them to arrive at the expected learning outcomes under the guidance of the facilitator. This projects the learner-centredness, small group activity, cooperation, and interactivity features of the PBL strategy (Gaudet et al., 2010; Reutova, 2012; & Weimer, 2002). The brainstorming and dependence on each other to deepen their conceptual understanding to be able to solve questions could not be done alone (Vygotsky, 1978) and their ability to express their understanding of the concept (Baroody & Coslick, 1993; Hulukati, 2005; Van de Walle et al., 2010) could contribute to the desired learning outcomes. Again, Table 2 revealed that the pre-service teachers performed abysmally in the topic "Ratio and Rates" in the pre-test with a mean score of 1.96 out of 5.0. The mean score in the post-test scores for the same pre-service teachers was 4.12. This suggests that with the help of the PBL intervention, the pre-service teachers improved upon their pre-test performance showing an improvement in learning outcomes. This observation cuts across the pre-post-tests for all eight lessons the authors taught. The observed improvement in learning with the PBL approach which aptly contained the tenets of the PBL concepts as conceptualised in the study supports the opinions of some scholars (Mughal & Shaikh, 2018; Padmavathy & Mareesh, 2013; Sin, 2006; Yadav et al., 2018; Zakaria et al., 2010;) that PBL has a positive effect on teaching mathematics and could increase learners' comprehension and aptitude to apply the concepts in everyday life.

Furthermore, the transcribed data from the focus group discussion in Table 4 helped to ascertain that there was an improvement in learning using the PBL approach. The teacher trainees indicated that they were able to solve more challenging questions in the group aside from being able to explain to their group members and the entire class. This indicates that they learnt and could share their experiences with their peers. This amplifies constructivism and the Zone of Proximal Development (Vygotsky, 1978) approach to learning as they concretize and share experiences in the learning process to cause a relative change in behaviour.

The outcome of the analyses based on data collected through a PBL disposition questionnaire showed that the pre-service teachers expressed a positive disposition toward the PBL approach in the learning of College of Education mathematics. Thus, the pre-service teachers expressed an appreciable level of *interest* in the PBL approach, a fairly strong *belief* that the approach could give a true learning outcome, and a positive *attitude* towards learning with the PBL approach. This aligns with a study conducted by McClean et al. (2006) on first-year medical students to ascertain their perception and values of small-group activity at the onset of a class. This revealed that the small-group activity of the PBL approach, turned out to project the personal development of the students, thereby, making them more tolerant, patient, self-

assured, and developing confidence in them. This opinion expressed by the pre-service teachers about the problem-based intervention supports mathematics learning among teacher trainees in Ghana. The results presented in respect of the two research questions suggest that there was an improvement in learning mathematics concepts which was evident in their improved learning outcomes and their positive disposition toward the PBL intervention.

Conclusion

The PBL strategy has proven to be impactful in the learning of mathematics concepts in Ghana among pre-service teachers. The study sought to examine pre-serve teachers' understanding of PBL and their application in learning mathematics concepts. The paired samples t-test was employed to establish the significance and subsequently the impact factor of the pre-post-tests employed to measure learning outcomes. Again, the study used the PBL disposition questionnaire to establish that teacher trainees had a positive disposition about the PBL approach. This culminated in ascertaining the impact of PBL on the learning of mathematics concepts among pre-service teachers.

These findings projected the features of PBL as conceptualised in the study. The findings further projected Lev Vygotsky's theory of constructivism and his ZPD concept underpinning the study. By this, the pre-service teachers could solve mathematics problems in groups with support from their peers. It can, therefore, be concluded that the introduction of the PBL approach by the authors helped to deepen the students' conceptual understanding of mathematics. This was evident in the improvement in the learning outcomes among the pre-service teachers. The study's outcome could also be influenced by the researchers' employment of a well-designed lesson plan that captured the independent variables of the proposed conceptual framework. Furthermore, the classroom configuration where the teacher trainees were put in small groups facilitated learner-centredness, cooperation, and interactivity which contributed to the success of the approach.

The methodology employed in the study was somewhat different from what was discovered in the pieces of literature consulted. Thus, in this study, the authors used an exploratory case study design involving 32 pre-service teachers and two college tutors. This made it different from the usual way of using quasi-experimental design involving the use of treatment and control groups to establish findings on related studies on PBL. Again, the proposed conceptual framework could serve as a contribution to knowledge and a source of information for further research. In a nutshell, the PBL intervention proved to be effective and desirable in the learning of mathematics concepts among the students of the case study group. This made the methodology somewhat different compared to what is discovered in related literature. The authors suggest that a longitudinal study using PBL could be conducted with a larger sample size to understand the direction of change over time.

Recommendations

The implications of the findings of the study could inform curriculum developers, policymakers, and stakeholders in the teacher education fraternity on best practices, policies, and future research on the PBL strategy. The following recommendations are made to inform policy, practice and future research:

a. PBL facilitators should occasionally reshuffle group members to allow them to share experiences and ideas with other groups.

b. Ghana Ministry of Education should incorporate a PBL in the mathematics curricula for pre-service teachers.

c. Educational researchers should conduct further research in other subject areas and at various educational levels to ascertain its effectiveness.

Limitations

Limitations in the study were bound to happen. These necessitated measures to give credence to the study outcomes but not to condemn the outcomes of the investigations. Using one intact class out of nine, and one college of education out of 48 could restrict the study outcomes from broad generalisation. However, using more than one intact class, and more than one college of education could have better established the impact of the PBL strategy. It could have also broadened pre-service teachers' disposition about the learning intervention for generalisation. The authors admit that a longer period than one month for the data collection could lead to thorough observation and critical examination of the PBL phenomenon for a more cogent outcome. Furthermore, the authors argue that the outcomes or observations from the study could have gone beyond just ascertaining whether the PBL approach could enhance the learning of mathematics concepts. It could have also ascertained students' perception of the instructional strategy if the approach had been compared to other instructional methods which the study did not.

Ethics Statement

The study, which involved pre-service teachers from Akrokerri College of Education, Ghana was approved by the authorities of the College. The consent of the students was accordingly sought.

Conflict of Interest

The authors declare no conflicting interests among them.

Authorship Contribution Statement

Boye: Conceptualization, design, analysis, writing. Agyei: Editing/reviewing, supervision.

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