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Teacher Conceptualization of Pedagogical Content Knowledge Through the Lens of Experts' Perspectives

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Abstract: This study compares experts' and teachers' conceptualization of pedagogical content knowledge (PCK). The study participants included teachers (n=20) enrolled in a graduate mathematics education course on PCK. Participants responded to two open-ended questions: a) describe in your own words what PCK is; b) provide an example of PCK. The responses were collected, qualitatively and quantitatively analyzed, and then compared to those suggested by experts to identify and describe the similarities and differences between teachers' and experts' conceptualizations using the Pareto analysis. Experts' and teachers' PCK components ranking was analyzed using the nonparametric Mann-Whitney U test. Even though the results of the quantitative analysis were not significant (e.g., the observed U-value is *32* whereas the critical value of U at $p < .05$ is *13*), the qualitative discussion on the differences between expert and teachers' ranking suggests insightful interpretation of priorities among PCK components across the two groups.

Keywords: *Expert perspective on PCK, graduate mathematics education, pedagogical content knowledge, teacher conceptualization of PCK.*

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Introduction

Pedagogical content knowledge (PCK) is essential in improving teacher quality. However, its definition is elusive and constantly changing, making it challenging to understand and use as a teaching model (Kind, 2009). This ambiguity has also made it difficult in teacher education to focus instruction on aspects that will increase teachers' PCK. This research will contribute to understanding how the expert conceptualizations differ from that of the teacher and will provide curriculum guidance for instruction in the teacher knowledge base.

As conceptualized by Shulman (1986), PCK was the missing paradigm that extended teacher knowledge beyond content to include knowledge of content representation, student misconceptions and difficulties, and instructional strategies (Abell et al., 2009). The gap in the current research on PCK lies in the limited understanding of how teachers themselves conceptualize PCK and how these conceptualizations compare to those of experts in the field. While extensive research has focused on defining and measuring PCK, less attention has been given to exploring the differences between expert and teacher conceptualizations of PCK. Understanding these differences is critical, as they can provide valuable insights into the challenges teachers face in developing PCK and inform more targeted and effective professional development. Moreover, the specific attributes that constitute PCK, and how these attributes manifest in teaching practice, remain underexplored. These gaps are significant because they directly impact the ability of teacher education programs to equip teachers with the PCK necessary to excel in diverse classroom settings.

Since its introduction, PCK has been researched extensively to determine its nature, yet despite general agreement that it is part of a teacher's expertise manifesting in improved teacher performance and student outcomes, scholars are hard-pressed to find a one-size-fits-all description (Copur-Gencturk & Tolar, 2022) in part because PCK is an elusive and hidden concept embedded in teachers' thought processes and often subconscious (Kind, 2009). Fernández (2014)

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confirms that the lack of consensus leads to confusion as researchers often need to specify the PCK model used in their research.

Developing a greater understanding of PCK attributes is relevant for teacher education. Therefore, the purposes of this study are:

1. to identify and describe the components of PCK most frequently cited by experts,
2. to describe teachers' conceptualization of PCK based on open-ended questions,
3. to compare and contrast the experts' conceptualization with those of the teachers participating in the study and
4. to make recommendations for PCK training within a graduate-level teaching course.

Responses to two open-ended questions asked of a cluster sample of teachers (n=20) enrolled in a graduate-level course called PCK in Teaching Mathematics at a public university in the U.S. Southwest will be compared to the experts' PCK conceptualizations.

Experts' Conceptualization of PCK in the Literature

In our systematic literature review focused on PCK conceptualization, we describe studies that review frequently cited PCK models and use these as a basis for our project. We used EBSCO, ProQuest, and JSTOR to identify peer-reviewed articles regarding PCK conceptualization. Next, we identify and describe standard components of PCK introduced by experts that will be used to organize and compare teacher conceptualizations to those of experts.

Since the introduction of PCK, academic experts have debated the nature of PCK, its relationship with content knowledge, instructional practice, student learning outcomes, personal characteristics, and its role in teacher training. Three meta-reviews of PCK by Kind (2009), Depaepe et al. (2013), and Fernández (2014) establish the framework for model selection and thematic identification of PCK components. In her study, *PCK in science education: perspectives and potential for progress*, Kind (2009) screens 500 research articles from popular databases such as ERIC and ZETOC with a publishing date starting at 1987 to select models that are theoretical, empirical, or that discuss changes in teachers PCK. From this selection, Kind (2009) provides an in-depth analysis of PCK models to gain a better understanding of how PCK is developed in science teachers to improve teacher quality. Fernández (2014) reviews major models of PCK and provides graphical representations of models used in the PCK literature. Although PCK is considered to be an essential knowledge base for the teaching profession there is still a lack of understanding and consensus about its definition. Fernández (2014) outlines and represents the most common models used saying that "since there are different ways of conceptualizing PCK and different authors propose different models in which some skills are prioritized over others, it is important to be aware and present which model is being used in research related to PCK" (p. 97).

Depaepe et al. (2013) set out to determine how PCK is conceptualized in mathematics education using ERIC, PsycInfo, and Web of Science research databases, they screened 811 articles selecting 60 research articles for inclusion in their study with findings about PCK conceptualization centered on six major research lines that include the nature of PCK, the importance of content knowledge, instructional practice, student learning outcomes, personal characteristics, and how PCK is acquired by instructors. These themes are met with significant disagreement, but generally, two of Shulman's (1986) core PCK components - knowledge of students' (mis)conceptions and knowledge of instructional strategies and representations – are considered essential (Depaepe et al., 2013). Despite differences in how PCK is conceptualized in different subject domains, Depaepe et al. (2013) identified the main PCK components in mathematics education research as knowledge of student mistakes and misconceptions, teacher knowledge of instructional strategies, content and subject knowledge, pedagogical and teaching knowledge, curriculum knowledge, assessment, context knowledge, and purpose and orientation. In what follows, we present detailed descriptions of the expert conceptualizations in the literature and synthesize these findings in a table showing the most often cited descriptions.

PCK Components

The confusion and lack of consensus regarding the definition of PCK necessitates elucidating components as defined by Depaepe et al. (2013). In what follows, we provide brief overviews of how experts conceptualized each component of PCK. We then provide a summary table that notes the frequency with which each component occurs across all studies reviewed. Following this analysis, we turn to our methodology and the findings of our research.

Knowledge of Students' Common Mistakes and Misconceptions: A feature of Shulman's (1986) PCK is that teachers must have "an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons" (p. 9). All models in this study considered that knowledge of students' common mistakes and misconceptions were a cornerstone of effective PCK with Shulman (1987), Grossman (1990), Marks (1990) adding that knowing *how* students understand certain aspects of the subject is also an aspect of PCK.

In addition to common mistakes and misconceptions, knowledge of students' goals and objectives (Fernández-Balboa & Stiehl, 1995), understanding of a students' previous knowledge, age levels, and abilities (Cochran et al., 1993), and knowledge of student experiences (Rollnick et al., 2008) all play a role in PCK. Magnusson et al. (1999) hold that teachers should have an understanding of the requisite knowledge for the lesson, previous knowledge students bring, as well as common misconceptions, especially those that run contrary to the lesson. Morine-Dershimer and Kent (1999) hold that through classroom communication, teachers will achieve greater understanding of the knowledge of learners which in turn, informs PCK. Carlsen (1999) reminds educators that in teaching students, conceptions are always changing in science. In knowledge of students' understanding, For An et al. (2004) knowledge of student thinking and misconceptions provides a starting point for instruction. At the same time, Alvidrez et al. (2024) emphasize the complex nature and role of teachers' framing of mistakes in students' learning. Rollnick et al. (2008) consider that knowledge of students and their experiences is one of the four main domains of PCK.

Hill et al. (2008) develop a framework in which knowledge of content and student (KCS) is essential for building lesson plans that take into consideration students thinking about math and for addressing common student errors; however, the measurement of KCS is not straightforward. Park and Oliver (2008) found that teacher's understanding of common misconceptions was a component of PCK, but also found that student questions increased teacher's understanding of misconceptions and increased a teachers' PCK.

Teacher Knowledge of Instructional Strategies & Representations: Shulman (1986, 1987) considers that the teacher must know "strategies that are fruitful in organizing student understanding" (Shulman, 1986, p. 10) recognizing that students bring previous knowledge and information into the classroom which must be taken into consideration when organizing instruction. Grossman (1990) and Carlsen (1999) note that instructional strategies are guided by the overall purpose of teaching the content, for example, in the teaching of *Hamlet,* if the purpose is to see its applicability to real world situations, the teacher will adopt a different strategy than if it is based on an historical perspective (Grossman, 1990). In contrast, Marks (1990) does not include purpose in the development of instructional strategies, rather, he holds that it is an awareness of students' misconceptions along with pedagogical knowledge, and content knowledge that helps a teacher select instructional strategies.

Different from the previous researchers above, Fernández-Balboa and Stiehl (1995) identify instructional strategies that are part of PCK. However, some of the strategies listed could easily be considered pedagogical knowledge or representations. Magnusson et al. (1999) view instructional strategies as one aspect of pedagogical knowledge. For subject related instructional strategies such as using cognitively demanding tasks in classroom, the teacher could draw on CK, PK, and context (Monarrez & Tchoshanov, 2022). Whereas topic specific instructional strategies including representations and connections could be drawn primarily from teachers' CK (Magnusson et al., 1999). Morine-Dershimer and Kent (1999) place instructional strategies as a subset of PK. Rollnick et al. (2008) include instructional strategies as part of PCK and their selection is indicative of a teacher's understanding of CK, PK, knowledge of students, and context. Park and Oliver (2008) find that instructional strategies that are subject and topic specific are instruments of PCK, with representations and activities a subset thereof, and their use is also affected by teacher efficacy and confidence in their use.

Shulman (1986) distinguished between instructional strategies and representations, but the line between the two has blurred over time and through various iterations on the concept of PCK. Shulman (1986) says that teachers that possess PCK will have the "most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations … [to make] that make [the subject] comprehensible to others" (p. 9). Shulman (1987) considers that representations include selecting the appropriate medium (examples and explanation) to transform knowledge for the student. Marks (1990) expands on Shulman (1986, 1987) to match activities and problem sets to students, as well as to use resources such as media and visuals as tools to represent instructional concepts. Magnusson et al. (1999) include representations as a subcomponent of topic specific instructional strategies. Rollnick et al. (2008) view representations as an indicator of how well a teacher has mastered the four domains of PCK – CK, PK, knowledge of students and context.

Content and Subject Knowledge: Content knowledge (CK) was not a component in Shulman (1986, 1987) or Grossman's (1990) conceptualization of PCK. Marks (1990) introduced CK as an element of PCK in his study of eight fifth-grade math teachers. Marks (1990) said that if PCK is the transformation of content knowledge towards teachability, then understanding the content or subject matter is non-negotiable because without this understanding it is difficult to highlight problem areas for students, select appropriate representations, and adopt instructional strategies. In their study of ten college professors, Fernández-Balboa and Stiehl (1995) model found that content is important and that it is ever-changing, therefore engagement and understanding of the content is essential. Carlsen (1999) concurs that content, especially in science, is constantly changing requiring teachers to remain current in their field.

Cochran et al. (1993) distinguished content knowledge for teaching as different from content knowledge for research, viewing PCK as dynamic and even modifying the name to PCKnowing to indicate its dynamism (Depaepe et al., 2013). Cochran et al. considered content knowledge as necessary for good teaching, finding that teachers who are asked to teach outside their discipline demonstrate less organization and are more prone to misconceptions when they do not know the

content. Morine-Dershimer and Kent (1999) identified content knowledge directly to PCK and indicated its relationship to curriculum knowledge. For An et al. (2004), content knowledge includes general and specific math knowledge for the grade level, while curriculum knowledge refers to understanding the *why* of the subject and selecting appropriate materials. In Rollnick et al. (2008) content or subject matter knowledge is a necessary component for PCK finding that when teachers do not know their content, they tend to become mechanistic, reverting to rote pedagogical tactics. Moreover, the lack of teacher content knowledge affects students' learning and understanding of mathematical concepts (Tchoshanov et al., 2019). In Hill et al. (2008), content knowledge is separated from PCK as an equal component of their reconceptualized model Mathematical Knowledge for Teaching (MKT) that includes common mathematics, specialized mathematics content, and mathematics at the horizon. For Park and Oliver (2008), content knowledge is a component of teacher efficacy.

Pedagogical and Teaching Knowledge: Shulman (1987), in his Model of Pedagogical Reasoning and Action (MPRA), adds pedagogical knowledge / knowledge of teaching as a component of PCK that involves the ability to comprehend, reason, transform, and reflect for instruction. For Carlsen (1999), pedagogical knowledge is interdependent with PCK and includes learners, classroom management, and curricular knowledge. Morine-Dershimer and Kent (1999) model of PCK provides more balance to the role of PK and considers that instructional strategies, reflection, context, classroom management and discourse all contribute to PK. For An et al. (2004) PK is termed knowledge of teaching and is the core element in the PCK definition including understanding students' thinking and learning. Rollnick et al. (2008) refer to PK as the best teaching practices based on research. For Hill et al. (2008), pedagogy is part of PCK and is labeled in their MKT model as knowledge of content and teaching.

Curriculum Knowledge: Grossman (1990) introduced the value of curricular knowledge as a concept in PCK, while still keeping content knowledge as a separate component of a teacher's knowledge base. Curricular knowledge, according to Grossman, allows the teacher to shape instruction, including certain elements and omitting others depending upon knowledge, interest, and values and importantly the purpose for teaching the content (Grossman, 1990). This is especially important when teaching abstract or difficult content. In her research, she tracked the teaching of Hamlet, noting that the PCK of the instructors was guided by how they viewed the play and how they were able to use their curricular knowledge to connect the concepts to students' lives (Comeaux, 1991). Carlsen (1999) agrees with Grossman (1990) placing subject specific curricula as a component of PCK.

Magnusson et al. (1999) included knowledge of science curricula saying that there are two types of such knowledge. The first is the knowledge of administrative curricular requirements while the second is knowledge of the subject's curriculum program, what instruction is appropriate at different age groups and how this relates to the instructor's educational orientation (Magnusson et al., 1999). Morine-Dershimer and Kent (1999) model of PCK positions curricular knowledge as informed by content knowledge, and then informing assessment with all three contributing to PCK. For An et al. (2004) curricular knowledge informs and is informed by PCK and contributes to teacher knowledge. Rollnick et al. (2008) also place curricular knowledge as an element of PCK indicating however, that content knowledge is needed to make decisions about which topics must be included or can be excluded from instruction. Although not a focus of their study, Hill et al. (2008) include curriculum knowledge as part of PCK. Park and Oliver (2008) draw from Grossman (1990) describing curricular knowledge as the understanding of salient topics and how to position these within instruction.

Assessment Knowledge: Magnusson et al. (1999) introduced knowledge of assessment as a component of PCK that includes identifying the essential, conceptual understandings within a learning unit and which methods of assessment are indicated for that concept. For Morine-Dershimer and Kent (1999) assessment is linked to the purpose and orientation of the teaching, presenting differing strategies for assessment noting that in the U.S. the strategy has been to use standardized tests, but other options exist. Rollnick et al. (2008) consider that knowledge of assessment goes hand in hand with knowledge of content because CK is needed to determine how to assess student learning. Park and Oliver (2008) consider knowledge of assessment a component of PCK stating the importance of knowing what to assess and how to assess.

Context Knowledge: For Shulman (1986), context knowledge was not considered to be part of PCK with issues of classroom management and organization held constant in his model. Context knowledge, however, was introduced as a component of PCK in Fernández-Balboa and Stiehl (1995) when college professors referred to student count, administrative support, availability of resources, the setting of the classroom, whether the classroom size is adequate, student attentiveness, and employment issues. Cochran et al. (1993) describe context knowledge as the knowledge of the political, cultural, and institutional influences that may affect teaching. However, for Carlsen (1999) context overarches and guides PCK based on the location of the class (whether rural or urban for example), relationships in the class, with the students, and parents including the broader educational environment as he reminds the reader of PCK's political roots to legitimize the teaching profession. Morine-Dershimer and Kent (1999) indicate that knowledge of specific and general context interacts and feeds directly into PCK. Content knowledge is one of the four knowledge types for PCK for Rollnick et al. (2008) and is essential to PCK. Rollnick et al. (2008) tested the significance of CK or subject matter knowledge by evaluating novice teachers with strong CK and experienced teachers in unfamiliar subjects finding that CK will impact *how* instruction occurs. It includes dimensions of class size, curriculum, and social considerations.

Teacher Knowledge of Purpose and Orientation: According to Grossman (1990), it is the *purpose* or the reason for teaching the content that overarches PCK encompassing curricular knowledge, the importance of understanding student misconceptions, and the use of instructional strategies. The purpose of the teaching, however, does not override PCK in Carlsen's conceptualization (Carlsen, 1999) as it does in Grossman (1990). According to Fernández-Balboa and Stiehl (1995) the purpose of the instruction for college professors embodies the teachers' values and beliefs including the teacher's commitment to the subject, its relevance in students' lives and their future. For Magnusson et al. (1999) purpose or orientation of instruction will influence how the teacher delivers the content. For example, if the goal is academic rigor, then students will be given difficult problems but if the goal is discovery, then activities will be student-centered and exploratory (Magnusson et al., 1999). Morine-Dershimer and Kent (1999) say that knowledge of purposes and ends are inseparable from assessment. Beliefs, which are categorized as purpose or orientation, guide PCK and instructional strategy according to An et al. (2004) depending upon whether the teacher is focused on learning a knowing or learning as understanding with the latter providing the preferred, long-term outcome.

Park and Oliver (2008) describe a teacher's self-efficacy in implementing strategies, knowledge acquisition and transmission are needed for effective PCK and indicate that this is measured by other researchers as the teacher's beliefs, or sense of purpose. In other words, teachers need to trust their PCK and although idiosyncratic, that comes with self-confidence and assuredness in their work. Cochran et al. (1993) indicate in their model that PCK is not static, is specific to the content, and increases with time and experience, noting that novice teachers are prone to making mistakes, but these are opportunities for reflection and growth.

A component that is new that does not align directly with purpose or orientation is that PCK is an experiential learning process for the teacher that happens during teaching and learning about students and is not taught or carried into the classroom by the teacher (Park & Oliver, 2008). Another component of PCK that does not quite fit into the category of knowledge of purpose or orientation is *reflection.* Shulman (1987) considered the act of reflection to PCK that includes "reviewing, reconstructing, reenacting, and critically evaluating" both the instructor and class performance (p. 15). Cochran et al. (1993) indicate in their model that for teachers to grow and increase PCK reflection is necessary, but it is not a component of PCK. For Morine-Dershimer and Kent (1999) reflection and personal beliefs about classroom practices and outcomes are part of a repeated process informing PK that is used in PCK. The interactive aspect of the Park and Oliver (2008) model of PCK embeds reflection at every stage as teachers reflect upon student misconceptions, the effectiveness of instructional strategies, and through this reflection, increase PCK.

The expert conceptualizations of PCK components are presented and tallied in Table 1, with the top identifying the PCK components. Each study reviewed is listed in the first column, and an x is placed by the conceptualized components included in that study. As seen in Table 1, by frequency count and the percentage totaled in the table's last two rows, experts consider *the knowledge of student mistakes and misconceptions* an essential component of PCK and a cornerstone in the studies reviewed. Knowledge of curriculum ranked second and was mentioned in 11 of the studies. Both knowledge of instructional strategies and representations and content knowledge were included in 10 of the studies reviewed. Having presented an overview of expert PCK conceptualizations and providing a summary of each, we now turn to discussion of the method that was used to analyze teacher conceptualizations and how they compare with expert conceptualizations.

Table 1. Synthesis of Literature Focused on the PCK Conceptualization

		PCK Components							
Study	**Discipline**	Mistakes & misconceptions	Inst. Strategies/ Representations	Content/ Subject	Pedagogy	Curriculum	Assessment	Context	Purpose or Orientation
Shulman (1986)	General	X	X						
Shulman (1987)	General	X	X		X				X
Grossman (1990)	Literature	X	X			X			X
Marks (1990)	Science	X	X	X		X			
Cochran et al. (1993)	Math	X		X	X			X	
Fernández-Balboa and Stiehl (1995)	Science	X	X	X				X	X
Carlsen (1999)	Science	X	X	X	X	X		X	X
Magnusson et al. (1999)	Science	X	X			X	X		X
Morine-Dershimer and Kent (1999)	General	X		X	X	X	X	X	X
An et al. (2004)	Math	X		X	X	X			X
Rollnick et al. (2008)	Science	X	X	X	X	X	X	X	
Hill et al. (2008)	Math	X		X	X	X			
Park and Oliver (2008)	Science	X	X	X		X	X	X	X
Abell et al. (2009)	Science	X	X			X		X	
Fernández (2014)	General	X		X	X	X	X		
Frequency Count		15	10	10	8	11	5	7	8
Percent of Total		20%	14%	14%	11%	15%	7%	9%	11%

Methodology

A survey research method was selected to describe and gain insight into how teachers (n=20) responded to two open ended questions:

1) Describe in your own words what PCK is.
2) Provide an example of PCK.

Teacher responses were qualitatively and quantitatively analyzed as to whether teachers' conceptualization of PCK differed from that of experts in the field. Teacher responses were compared to the components considered to be part of PCK as identified by experts. Following, we discuss participants, data collection, and data analysis.

Participants

The teachers that participated in the study were enrolled in a graduate-level class on PCK in the Mathematics Classroom at one of the U.S. universities located in the southwestern border region. The target audience of the course was middle school mathematics teachers. There were 14 females and six males in the sample. The ethnicity of most participating teachers was Hispanic (17) and White (3). Teachers also varied in teaching experience: from one year to 12 years. The survey was given to participants at the beginning of the class as a prompt to assess their prior knowledge about PCK. Participants had 20 minutes to respond to the survey: 10 minutes per question. Confidentiality was maintained by assigning each of the 20 participant's responses a number.

Data Collection

The self-completion open-ended survey questions were presented at the beginning of an instructional session without any prior prompting with the intent of collecting participants' previous knowledge of PCK. The first question asks respondents to describe PCK in their own words allowing for broad brushstrokes in the response. The second question drills deeper to extract examples and serves to solidify the researcher's understanding of each participants' PCK. The survey questions were designed to answer the key research questions about how teachers conceptualize PCK by asking for its description and an example (See Appendix A). There were 20 survey responses per question that were then compiled and prepared for analysis.

Data Analysis

Responses to each interview question were uploaded to Taguette, a free, open-source text tagging tool with data analysis consisting of coding teacher responses to the survey questions, conducting a frequency analysis, and then a Pareto analysis to identify the most prevalent teacher PCK conceptualizations. We used the Mann-Whitney U test to compare teacher vs. expert ranking on PCK components. It is a nonparametric test that compares two independent groups (e.g., experts and teachers) without assuming that values are normally distributed. To accurately apply and interpret the U test results, we closely followed the main requirements of the Mann-Whitney statistic technique, which are the following: two random, independent samples; the data is continuous; the data should be ordinal, interval or ratio; there should be no ties (though the U test has a way to handle ties).

Data Coding

The qualitative data were analyzed using meaning coding techniques (Kvale & Brinkman, 2009; Saldaña, 2015). Two raters independently coded teacher responses on both questions: inter-coder agreement for question 1 was at r=0.75, for question 2 – at r=0.67. This level of agreement is considered adequate for educational research (Hill et al., 2008). Interview responses were analyzed using meaning coding on a word by word basis for each of the participants' survey responses. As an example, when asked to respond to the following statement, Describe in your own words what is PCK, Participant 1 responded as follows: PCK is understanding content at a very deep level to be able to teach in different ways to accommodate different levels of students with different learning styles.

The following demonstrates underlined coded words with codes in brackets.

PCK is understanding content [Content Knowledge] at a very deep level [Content Knowledge, Understands the Why] to be able to teach [Pedagogy] in different ways [Representations] to accommodate different level [Accommodative] [knowledge student understanding, Cares for student, Accommodative] of students with different learning styles [knowledge student understanding].

Each survey response was coded in this manner. The survey questions were analyzed using linguistic analysis and meaning coding to interpret the inherent meaning of the responses (Kvale & Brinkman, 2009) that would organically coalesce into experts' components of PCK. Coding results were tallied within each PCK component, and the codebook was developed (Appendix B). These codes were then categorized based on how they fit experts' conceptualizations of PCK (see Table 1 for details on their conceptualizations). A frequency analysis was conducted for the number of teacher codes that fit the experts' conceptualizations and then sorted in descending order. A Pareto analysis was used to organize the codes and help address the research question of how experts' and teachers' conceptualization of PCK compare to each other. The following section describes the Pareto analysis.

Pareto Analysis

A Pareto analysis is based on the Pareto principle, where it is estimated that 80% of an outcome or phenomenon may be accounted for by 20% of the possible causes. A Pareto analysis organizes data and allows researchers to identify important components of PCK. However, this may vary depending on the phenomena studied, allowing the researcher to focus on solving the root cause of said event or phenomenon (Alkiayat, 2021). The frequency of occurrences, events, or, in the case of this study, components are organized in descending order and weighted by the cumulative percentage of the occurrences as they impact the phenomena. Graphical representation of the Pareto analysis consists of a bar chart where each contributing factor, or PCK component, is represented as a bar on the x-axis, the frequency count for each component is represented on the y-axis, and their cumulative percentage is represented on the z-axis. The graphing feature in Excel was used to create the Pareto charts.

In the charts, an 80% cutoff line is established where components to the left of the 80% are deemed vital to the phenomena, and those components to the right of the cutoff of the line are deemed trivial components (Clinical Excellence Commission [CEC], n.d.). The Pareto analysis is often used in industry but has also been used in qualitative studies to organize and evaluate coded data (Abbas et al., 2020; Gupta & Shrivastava, 2023; Zinebi et al., 2018). The Pareto analysis allows the researcher to focus on the vital few components that contribute to the phenomena rather than the trivial many (CEC, n.d.).

Findings

PCK Components: Aggregated Expert Perspective

The expert rankings were determined by reviewing each study to see how the authors or experts conceptualize PCK. The researchers categorized and tallied the experts' conceptualization using the main PCK components identified by Depaepe et al. (2013), as seen in Table 1, which are knowledge of student mistakes and misconceptions, teacher knowledge of instructional strategies, content and subject knowledge, pedagogical and teaching knowledge, curriculum knowledge, assessment, context knowledge, and purpose and orientation. Expert PCK conceptualization is tabulated in Table 2 and

demonstrates the frequency that PCK components appear across experts' studies. Table 2 also shows the percentage that each component represents as a percentage of the total conceptualizations, as well as the cumulative percentage.

Table 2. Frequency, Percentages, and Cumulative Percentage of Expert PCK Conceptualizations

Conceptualization of PCK		Frequency	Percentage of Total	Cumulative Percentage
1	Student Mistakes & Misconceptions	15	20%	20%
2	Curriculum Knowledge	11	15%	35%
3	Inst. Strategies & Representations	10	14%	49%
4	Content / Subject Knowledge	10	14%	62%
5	Pedagogy/ Teaching Knowledge	8	11%	73%
6	Purpose and Orientation	8	11%	84%
7	Context Knowledge	7	9%	93%
8	Assessment Knowledge	5	7%	100%
	Total	74	100%	

Pareto Analysis of Expert Perspective

The Pareto analysis indicates that there is a diversity of expert conceptualization about what constitutes PCK. As seen in Figure 1, the 80% cutoff line occurs between five to six PCK components, indicating that between 62 to 75% of the components account for 80% of the conceptualizations. In a Pareto analysis normally 80% of PCK conceptualization would come from 20%, or two, of the components. The Pareto analysis indicates no clear consensus of expert PCK conceptualization.

Knowledge of students' common mistakes and misconceptions, as conceptualized by Shulman (1986), appears in all studies and is one area of agreement between experts. Another area of agreement is curriculum knowledge included in eleven of the studies reviewed, indicating that teacher flexibility in shaping the course, understanding which topics are the most important, and understanding the resources available for instruction (Grossman, 1990) is a vital conceptualization of PCK. Content knowledge and instructional strategies and representations were conceptualized in ten models. Both pedagogical knowledge and purpose/orientation were included in eight models, and either could fall within the 80% cutoff. Knowledge of context was mentioned seven times, and assessment knowledge was mentioned five times. As shown in Figure 1, purpose or orientation, context knowledge, and assessment knowledge are not as strong contenders for PCK conceptualization.

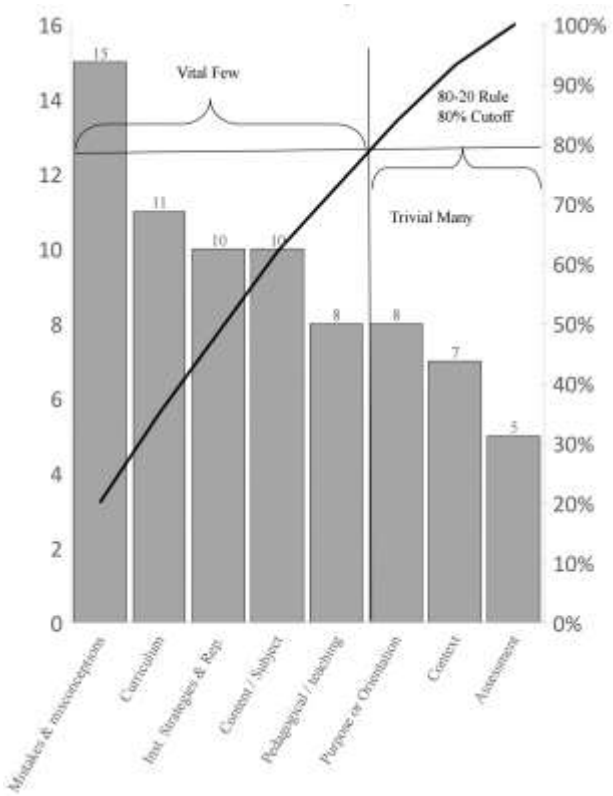

Figure 1. Pareto Analysis: Expert PCK Components

The use of the Pareto analysis to identify the top contenders for PCK conceptualization did not apply in this case. However, it does confirm a diversity of opinions on what PCK is and how it can be conceptualized. In the next section, we analyze teacher responses using the same Pareto analysis approach to compare whether teacher conceptualization mirrors that of experts.

PCK Components: Aggregated Teacher Perspective

This research rests on two open ended questions that allow for richer responses and indicates to the researcher what is on the respondents' minds (Popping, 2015), however, the open-ended questions also present some problems in analysis because of the difficulty in making sense of the diversity of answers, or the lack of a response leaving much to interpretation and meaning coding. The method for coding responses was presented in the previous data analysis section of this paper. In the following sections, we present the synthesis of participating teachers' responses to two survey questions. Responses were coded and then grouped into the main PCK component areas cited by experts. A Pareto analysis is conducted for each question and then compared to experts' PCK conceptualization.

Question One. Describe in your own words What PCK is?

Participants provided their definitions of PCK, and phrases were coded by the researcher and assigned to the categories outlined by Depaepe et al. (2013). Teachers' perspectives on PCK emphasized the need to accommodate students, provide multiple ways of learning, and understand their content and subject. For example, one participant states that:

> *I understand PCK is the teacher's knowledge of teaching along with what they teach. A method the teacher makes the matter accessible to students. In other words, different techniques to ensure the student comprehend the content. The teacher is required to know how to integrate pedagogy, and content to create the right environment for every student.*

Codes that emerged are indicated in the next passage where key words are underlined and the codes are in brackets:

> *I understand by PCK, as the teacher's knowledge of teaching [Pedagogy] along with what they teach [Content Knowledge]. A method [Pedagogy] the teacher makes the matter accessible to students [Accommodative, Cares for student]. In other words, different techniques [Pedagogy, Cares for student, Accommodative] to ensure the student comprehend the content [Student outcomes]. The teacher is required to know how [Teacher knowledge] to integrate pedagogy [Pedagogy], and content [Content Knowledge] to create the right environment for every student [Accommodative, Cares for student].*

This analysis was conducted across all participant responses with Table 3 totals the number of codes that fall within each of the PCK components. The frequency of the codes assigned for each of the 20 participant responses is arranged in descending order. The data in Table 3 and the Pareto analysis in Figure 2 show that 80% of the teacher conceptualizations are spread across four vital PCK components: instructional strategies and representations, content and subject knowledge, pedagogical and teaching knowledge, and knowledge of student mistakes and misconceptions.

Table 3. Question 1 - Frequency, Percentages, and Cumulative Percentage of Teacher PCK Conceptualizations

	Conceptualization of PCK	Frequency	Percentage of Total	Cumulative Percentage
1	Instructional Strategies & Representations	62	25%	25%
2	Content and Subject Knowledge	53	21%	46%
3	Pedagogical and Teaching Knowledge	44	18%	64%
4	Student Mistakes and Misconceptions	33	13%	77%
5	Purpose and Orientation	18	7%	84%
	Assessment Knowledge	17	7%	91%
7	Curriculum Knowledge	13	5%	96%
8	Context Knowledge	10	4%	100%
	Total	250	100%	

The data in Table 3 is used to develop a Pareto chart as shown in Figure 2. Four PCK components or 50% of components account for 80% of teacher PCK conceptualizations with teachers ranking instructional strategies and representations as central to their conceptualization of PCK Teachers also consider that content/subject knowledge and pedagogical knowledge are important components of PCK, as is an understanding student mistakes and misconceptions The remaining components: purpose and orientation, assessment knowledge, curriculum knowledge, and knowledge of context are the trivial few and do not resonate as definitions of PCK for the participants.

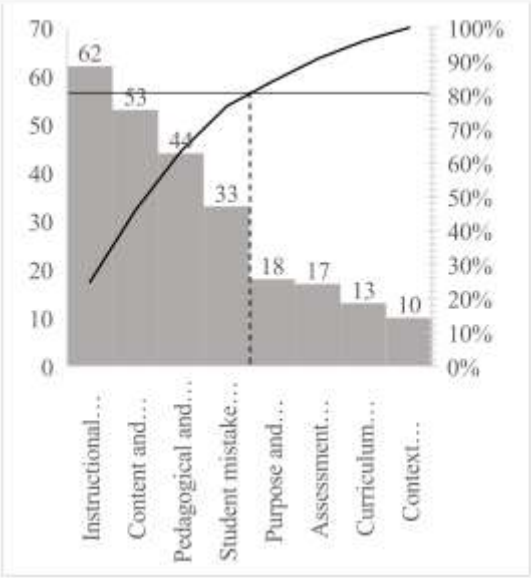

Figure 2. Pareto Analysis: Teacher Perspective, Question 1

In their descriptions of PCK, teachers used phrases such as *using building blocks* and *finding different ways of explaining* that were coded as instructional strategies and representations, for example. Participants indicated that PCK is content and subject specific and includes an understanding of how and why a subject is organized in a certain manner. For this study, participants used words that indicated a certain level of awareness about ensuring student learning, using words such as *grasp, understand, and comprehend* lessons when referring to students' learning.

In comparing teacher perspectives on PCK to expert perspectives, we found that teachers ranked instructional strategies and representations more often in their descriptions at approximately 25% frequency, in sharp contrast to the 14% or 10 studies that include instructional strategies and representations as part of PCK conceptualization. Further content and subject knowledge comprised 21% of the coded words, indicating that teachers believe the content and subject knowledge are critical. In contrast, experts included it in only 14% of the studies. However, one factor that may account for this difference is that experts may assume that content and subject knowledge are essential for instructors' instruction. Pedagogical teaching and teaching knowledge ranked third at 18% compared to 11% and expert instructions. In comparison, knowledge of student mistakes ranked at 13 percent compared to the 20% indicated by experts. The analysis of codes indicates that teachers think about PCK differently from the conceptualizations expressed in expert studies.

Question 2: Provide an example of PCK

In question 2, teachers are asked to provide an example of PCK. Teacher examples of PCK were centered on components of content and subject knowledge, instructional strategies and representations, and knowledge of student mistakes and misconceptions, as indicated in Table 4. The ranking of remaining components are pedagogical and teaching knowledge, purpose and orientation and assessment, curriculum, and context knowledge.

Table 4. Question 2 - Frequency, Percentages, and Cumulative Percentage of Teacher PCK Conceptualizations

	Conceptualization of PCK	Frequency	Percentage of Total	Cumulative Percentage
1	Content and Subject Knowledge	101	30%	30%
2	Instructional Strategies & Representations	79	24%	54%
3	Student Mistakes and Misconceptions	69	21%	74%
4	Pedagogical and Teaching Knowledge	48	14%	88%
5	Purpose and Orientation	19	6%	94%
6	Assessment Knowledge	15	4%	99%
7	Curriculum Knowledge	3	1%	99%
8	Context Knowledge	2	1%	100%
	Total	336	100%	

When asked to provide an example of PCK, respondents used more phrases and words that were tied to content knowledge. For example, one participant responded:

> *PCK is an understanding of material beyond is usefulness in the classroom, and how to present it to students. An example is teaching slope beyond students using rise over run to calculate. How does the slope of the graph affect the data? What can the slope inform about the data? How is the slope of the data relevant to us?*

Codes that emerged are indicated in the next passage where key words are underlined and the codes are in brackets:

> *PCK is an understanding of material [Content Knowledge] beyond is usefulness in the classroom [Curriculum knowledge], and how to present it to students. [Pedagogy, Representations, knowledge student understanding] An example is teaching [Pedagogy] slope [Subject Specific, Content Knowledge] beyond [Curriculum knowledge] students using rise over run to calculate [Subject Specific, Content Knowledge]. How [Understands the Why] does the slope [Subject Specific, Content Knowledge] of the graph [Content Knowledge, Subject Specific] affect the data [Content Knowledge, Subject Specific]? What [Understands the Why] can the slope [Content Knowledge, Subject Specific] inform about the data [Content Knowledge, Subject Specific]? How is the slope [Subject Specific, Content Knowledge] of the data [Subject Specific, Content Knowledge] relevant [Understands the Why] to us?*

In the examples, participants made direct reference to math concepts using words such as slope, equation, and data. Participants also indicated that real world connections and use of the subject beyond the classroom was important, indicating that curricular knowledge is an element of content knowledge. In providing examples of PCK, respondents drew on language that indicated an increased use of representations, technology and media, plus a need to teach students efficiently using available teaching materials and methods. Several responses focused on the need to be accommodative, with an eye to student outcomes. Responses were student centered and indicated a level of care for the student.

As seen in Figure 3, three PCK components: content and subject knowledge, instructional strategies and representations, and student mistakes and misconceptions manifested in the examples that teacher participants provided in their responses more closely matching expert conceptualizations.

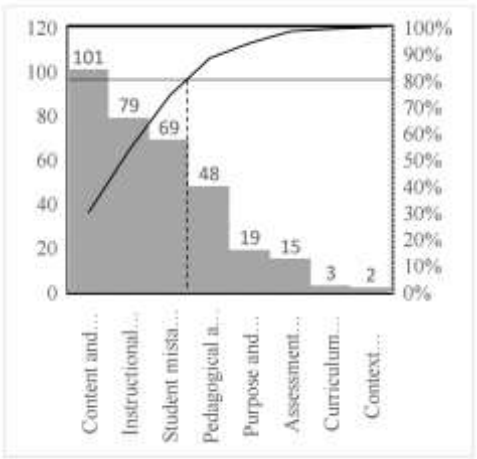

Figure 3. Pareto Analysis: Teacher Perspective, Question 2

When asked to provide examples of PCK, the teachers used words and expressions that were more aligned with content and subject knowledge, which they ranked as the most critical component of PCK based on the coding of their responses. Contents and subject knowledge for experts ranked at 13.5%; however, content and subject knowledge may be a given assumption within experts' conceptualizations. The second essential component was the teacher's knowledge of instructional strategies and representations, followed by knowledge of student mistakes and misconceptions. Based on the partial analysis, these three were the vital few components that comprised 80% of teachers' conceptualizations when asked to provide examples of PCK. The remainder of pedagogical and teaching knowledge assessment knowledge, context knowledge, teacher knowledge of purpose and orientation, and curriculum knowledge are ranked as trivial few within the Pareto analysis.

Comparison Between Expert and Teacher PCK Conceptualization

Experts consider that knowledge of student mistakes and misconceptions is the most essential element for a teacher's PCK and is present in each of the studies reviewed; however, when asked to describe PCK, teachers indicated that PCK was comprised of instructional strategies and representations and content and subject knowledge, ranking both components first and second in importance while experts ranked these in third and fourth place. In contrast, experts ranked curriculum knowledge as second in importance, while teachers ranked curriculum knowledge seventh in order of importance. Pedagogical and teaching knowledge and knowledge of students' mistakes and misconceptions ranked

third and fourth in order of importance. Teachers and experts gave lower priority to knowledge of assessment and context. However, the qualitative discussion on the differences between expert and teachers' ranking that we address below deserves close examination and suggests insightful interpretation of priorities among PCK components across the two groups.

Table 5. Comparing Expert and Teacher Ranking of PCK Components

Rank	Expert Perspective	Teacher Perspective: Question 1	Teacher Perspective: Question 2
1	Knowledge of student mistakes and misconceptions	Knowledge of instructional strategies & representations	Content and subject knowledge
2	Curriculum knowledge	Content and subject knowledge	Knowledge of instructional strategies & representations
3	Knowledge of instructional strategies & representations	Pedagogical and teaching knowledge	Knowledge of student mistakes and misconceptions
4	Content and subject knowledge	Knowledge of student mistakes and misconceptions	Pedagogical and teaching knowledge
5	Pedagogical and teaching knowledge	Knowledge of purpose and orientation	Knowledge of purpose and orientation
6	Knowledge of purpose and orientation	Assessment knowledge	Assessment knowledge
7	Context knowledge	Curriculum knowledge	Curriculum knowledge
8	Assessment knowledge	Context knowledge	Context knowledge

We performed the Mann-Whitney U test to compare the expert and teacher ranks on the main PCK components. Both for questions 1 and 2, the expert vs. teacher comparison yielded the U-value at *32*. Whereas the critical value of U at $p < .05$ is *13*. The z-score is *0.05251*, at $p=.96012$. Therefore, the result is not significant. We are cognizant that the test is sensitive to a small sample size of ranks in a sense that the approximation to the form of the normal distribution becomes less robust at sample size of ranks is smaller than *10*.

However, the qualitative discussion on the differences between expert and teachers' ranking that we address below deserves close examination and suggests insightful interpretation of priorities among PCK components across the two groups.

Discussion

The purpose of this study was to identify and describe the components of PCK most frequently cited by experts and then, based on data collected from teachers, describe their conceptualization of PCK. These would then be compared and recommendations for PCK training would follow. Two themes emerging from both the first and second research question was that teachers think that instructional strategies and representations and content and subject knowledge are the leading components of PCK, which contrasts with the experts' perspective that knowledge of students' common mistakes and misconception is the primary indicator of PCK followed by knowledge of curriculum, and then content and instructional strategies and representations. We next discuss teachers' top PCK conceptualization, how they compare to experts' conceptualizations, and their implications.

Knowledge of Students' Common Mistakes and Misconceptions

The frequency with which *Knowledge of Students' Common Mistakes and Misconceptions* appears for the experts is 20 percent compared to 12 percent for the teachers. Participant #9 discussed misconceptions and how to navigate them by stating that teachers "acquire more and more knowledge and experience on how they need to teach certain material in order for students to properly and successfully learn," while participant #10 explained that PCK was the "ability to understand misconceptions." The remainder of the words tagged under this component indicated that the instructor understands the need for students to grasp the material, the need to understand previous knowledge, and to accommodate learning styles. The examples provided in question two focused on the need to be accommodative, with an eye on student outcomes.

Responses were student centered and indicated a level of care for the student. In contrast, the experts positioned knowledge of the students' common mistakes and misconceptions at the center of PCK as did Shulman (1986). One implication from this finding is that it suggests that teachers do not give this concept as much emphasis as experts do. Therefore, one recommendation for training teachers is to have them investigate common students' mistakes and misconceptions in their discipline either as research (Alvidrez et al., 2024) or a case study as advocated for by Shulman (1986).

Instructional Strategies and Representations

In ten out of the fifteen studies reviewed, experts identified instructional strategies and representations as a component of PCK with the term comprising fourteen percent of the total PCK components. Several responses referred to strategies, methods, and representations. For this study, respondents used words that indicated a certain level of awareness about ensuring student learning. There were thirty-six tagged phrases and/or words that referred to instructional strategies, representations, and technologies were included. For example, participants #11 referred to "the use of technology, the use of calculators in the classroom" and participant #12 stated "I'm currently using hands on manipulatives (balance beams) to tie together equations and the idea of balance in general. Providing information and a visual to understand the idea of balancing equations." Each referred to the tools and manipulatives in their responses with other participants expressing the need to find methods to explain content differently and to be culturally responsible, for example. In contrast, the experts view instructional strategies as a method of organizing student learning and understanding (Monarrez & Tchoshanov, 2022; Shulman, 1986) using overall goals and purpose to set the course (Grossman, 1990). The study findings suggest that both experts and teachers indicate the need to reach the student through representations.

Content/ Subject Matter Knowledge

Teachers indicated that content and subject matter knowledge is an important component of PCK, ranking it as the third most important component when responding to question 1, and ranking first when asked to provide an example of PCK in question 2, as can be seen in Figures 2 and 3 respectively. Content knowledge includes subject specific knowledge and an understanding of the *why* of the subject (Tchoshanov et al., 2019). For example, participant #15 describes PCK in question one as "[It is] How well you know your subject and also how to teach it." In responding to the second question, teachers responded with more direct references to mathematics concepts indicating that real world connections were important saying that "[PCK]… is not only delivering the content like a textbook but using manipulatives, activities and maybe even real-life experiences to connect knowledge to students."

Prioritization of content knowledge stands in sharp contrast to the seven studies or 14 percent of studies that include it as a component. In these studies, experts found that teachers who were not secure in their content knowledge demonstrated poorer classroom performance (Cochran et al., 1993). Thus, one of the main implications of the study is that for the new teacher who may still be insecure in his or her content knowledge, content and subject matter training may be more impactful in developing PCK and help teachers acquire PCK more quickly.

Pedagogical and Teaching Knowledge

Pedagogical and teaching knowledge appeared in eight studies and ranked fifth in the number of times the concept appeared as a component of PCK. In contrast, in responding to question 1, pedagogical and teaching knowledge ranks as the third most important contributor to teacher PCK conceptualization. For example, participant #9 describes PCK as "...what teachers know about teaching and applying it to what they know about what they teach." In responding to question 2 that asks for an example of PCK, participants #6 responded "An example would be designing a lesson on quadratic equations and using the flight path of a football to describe the shape of the equation, as well as what certain coordinates would mean. For example, where the equation equals zero would be either the starting point of where the ball is thrown and where it hits the ground. The maximum of the equation would be the max height of the ball." For teachers, pedagogical and teaching knowledge is key component of PCK using phrases such as *content area you can teach*, *effective lessons*, and *how to teach.* The implication of these findings is that experts may want to consider increasing their research on the role of pedagogy and teacher knowledge in developing teacher PCK (Copur-Gencturk & Tolar, 2022).

Curriculum Knowledge

Curriculum knowledge refers to the teacher's ability to shape instruction depending upon knowledge and interest for the purpose for teaching the content (Grossman, 1990). Curriculum is referenced by eleven of the studies reviewed as having a direct connection to PCK. Curriculum for the experts implies content knowledge to know which elements are essential for organizing instruction. Teachers do not place as much emphasis on curriculum. The potential question for follow up research then becomes "why" don't teachers place emphasis on curriculum although the experts deem that curriculum helps to guide the learning through course materials and progression to training.

Conclusion

The purpose of this study was to help clarify the ways in which teacher and expert conceptualizations differ from one another and to offer curriculum recommendations for teacher instruction. The research set out to identify and describe the components of PCK most frequently cited by experts, to describe teachers' conceptualization of PCK based on open-ended questions, to compare and contrast the experts' conceptualization with those of the teachers participating in the study, and to make recommendations for PCK training within a graduate-level teaching course.

The first recommendation is that if we would like to close the gap in PCK understanding, teachers need to focus on addressing students' common mistakes and misconceptions. As suggested by Shulman (1986) and others, this could take

the form of a case study, focused instruction dissecting common students' misconceptions through video or in the actual classroom focused on practice identifying problem areas. Second, teacher training should include an increased focus on the curriculum as a practice that will provide the teacher flexibility to shape the course. The third component – Instructional Strategies and Representations – seems to be one of the consensus points between experts and teachers and is important because it is connected to areas of teacher efficacy (Park & Oliver, 2008) and classroom performance. Study participants seemed to feel this was more important than some of the other components (e.g., curriculum).

Recommendations

The practical recommendation that could be drawn from this study is improve teacher professional development in unpacking and understanding main components of PCK and their role in student learning. Notwithstanding, the significance of the study is that it provides valuable insights and demonstrates the disconnect between the experts and the teachers' priorities in PCK conceptualization and its components. The practical implication of the study is that it could assist those who provide teacher training to develop the curriculum focusing on those PCK components that will yield the most benefit to teachers taking into consideration the contexts and needs of the schools where they are teaching.

Limitations

There are several limitations and weaknesses with this study. First, the contextual knowledge of the participants was limited to the demographics presented in the study. Second, if researchers had an opportunity to replicate the study (e.g., future research) they would like to have an opportunity for a follow up interview with a selected number of participants. Third, coding for this study was done in three stages with the use of code emerging from the text of the respondents and from the researchers' interpretation which may have introduced bias into the study. Fourth, the findings from the small sample size of twenty participants means that findings cannot be generalized to other populations. And finally, other models of PCK could also be introduced which may yield different results.

One of the directions for future research as kindly suggested by the esteemed reviewers could be an investigation of the differences in PCK conceptualization between early career and experienced teachers.

Ethics Statement

Since the participants were graduate level students fulfilling the assignments of the course they were enrolled in, the study was waived by the Ethics Committee Approval.

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Conflict of Interest

There is no conflict of interest involved.

Authorship Contribution Statement

Gamez: Data analysis, drafting manuscript, critical revision of manuscript, statistical analysis. Tchoshanov: Concept and design, data acquisition, data interpretation, critical revision of manuscript, supervision, final approval. Villalobos: Data analysis, drafting manuscript, manuscript revision, technical support.

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Appendices

Appendix A: Summary of Survey Responses

Summary of Survey Responses to Question 1 and 2

	Question 1: Describe in your own words what Pedagogical Content Knowledge is.	Question 2: Provide an example of Pedagogical Content Knowledge below.
Participant1	PCK is defined as a deeper understanding of teaching content and a need for flexibility.	Student centered teaching techniques are referenced with no reference to teacher PCK in classroom.
Participant 2	Defines PCK as the linkage between content knowledge and instructional methods.	No example of PCK yet describes the teacher as the mediator for student success through methods and strategies. There is no reference to teacher PCK in the classroom.
Participant 3	PCK is viewed as a skill that is studied and practice.;	No example of PCK is provided, rather the participant draws on personal learning experiences in her/his field.
Participant 4	Understands the concept of PCK and its significance in helping students acquire knowledge for use in real life.	No example of PCK is provided, but rather an expansion on the definition of PCK.
Participant 5	No definition of PCK is provided, rather a concern for her incomplete teacher training. Indicates an understanding of curricular knowledge and the need for effective presentation of content.	Uses teaching of slope beyond the rise over the run as an example.
Participant 6	The PCK definition is not concrete, but rather, what someone might infer from the term pedagogical content knowledge.	Concrete example provided using flight path of a football to describe quadratic equations.
Participant 7	The respondent seems to have a cursory understanding and seems to be developing his or her own understanding in trying to answer this question.	No example is provided but mentions knowledge of students, knowledge of strategies which are based on research.
Participant 8	Respondent used the meaning of the words "Pedagogical Content Knowledge" to discern meaning and to write a response.	No example was provided, but this respondent understands the importance of using different strategies to help students grasp concepts and remaining flexible.
Participant 9	Definition of PCK includes experience gained over the years and the teacher's ability to teach in an efficient manner.	A discussion of misconceptions and how to navigate them is used as an example.
Participant 10	Respondent defines PCK as the capacity to teach, creativity, and desire to teach with a teacher centered perspective as in "filling an empty vessel."	No example is given, but a further explanation of PCK as the understanding of student misconceptions and the need for teacher flexibility.
Participant 11	Definition of PCK refers to technology and the integration of knowledge and tools.	Strategies are provided as examples as well as the use of balance beams and manipulatives.
Participant 12	Defines PCK as the combination of pedagogical and content knowledge.	No example provided. Uses the example of manipulatives such as balance beams and manipulatives communicate concepts.
Participant 13	Defines PCK as the blending of pedagogical knowledge and content knowledge using the existing structures as a basis for developing and leveraging instruction.	No example is provided but there is a discussion of the need to use problem solving techniques as a possible instructional strategy and also looks to the creation of an environment of inquisitiveness and fun.
Participant 14	The definition of PCK is incomplete referencing curriculum, but there is no discussion of the teacher - student relationship.	No example is provided, but respondent further defines PCK as part of a Vygotsky construct.

Summary of Survey Responses to Question 1 and 2

	Question 1: Describe in your own words what Pedagogical Content Knowledge is.	**Question 2: Provide an example of Pedagogical Content Knowledge below.**
Participant 15	The shortness of this response indicated that the interviewee did not have sufficient knowledge to answer the question.	There is no example provided of PCK, but rather some of the strategies that might be used in terms knowledge of teaching strategies.
Participant 16	PCK is defined by teachers using the tools available to make learning accessible. A focus is also on flexibility needed to have competent classroom discussion.	An example is provided that combines knowledge of the concept and use various interpretations to teach effectively with students.
Participant 17	PCK is defined as a method of teaching that engenders student interest.	The example provided is the use of tools to facilitate teaching, but there is no indication of an understanding of what makes things difficult for students, or how representations can be used.
Participant 18	Defines PCK as the integration as pedagogy and content.	The example of PCK draws on teaching techniques.
Participant 19	PCK requires an in-depth understanding of the content, stressing the importance of mastering the underlying fundamentals of the subject.	As an example, respondent refers to personal learning experiences to situate self in the students' space.
Participant 20	Respondent defines PCK as the teacher being technically competent and having the methods needed for the field of study.	No example is provided, but respondent refers to the use of manipulatives.

Appendix B: Codebook

Codes	**Teacher responses**
Accommodative	"Knowledge of the strategies most likely to be fruitful in reorganizing the understanding of learners, because those learners are unlikely to appear before them as blank slates" (Shulman, 1986, p. 9 -10)
Cares for Student	Indicates that student needs are important
Cognitive Perspective	"Category of teacher's knowledge base, typically define in line with Shulman..." (Depaepe, 2013, p. 9). It is static and can be taught.
Combine	Combine is for references that speak to the blending of CK and PK
Content Knowledge	This refers to "the amount and organization of knowledge per se in the mind of the teacher" (Shulman, 1987, p. 9).
Critical Thinking Skills	Focused on student critical skill development
Curriculum Knowledge	Knowledge of the usefulness of the topic and what is taught in other subjects. Knowledge of other curriculum materials (Shulman, 1986, p. 10).
Experience	Teacher experience in the classroom
Misconceptions	Knowledge of Misconceptions: "teachers' knowledge of common conceptions, misconceptions, and difficulties held by students regarding particular subject matter" (Dapaepe et al., 2013, p. 17).
Pedagogy	"knowledge for teaching ...the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations..." (Shulman, 1986, p. 9)
Technology & Media	Use of media and technology as an element of PCK
Knowledge of student understanding	Aware of how students conceptualize new knowledge.
Previous Knowledge	This might be under Content Knowledge as well, since Shulman (1986) says that content knowledge is "The teacher need not only understand that something is so; the teacher must further understand why it is so... (Shulman, 1986, p. 8). Knowledge of student understanding is One of the four components of PCK. Dapaepe, 2013, p. 13).

Appendix B: Continued

Codes	Teacher responses
Representations	"...the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations-in a word, the ways of representing and formulating the subject that make it comprehensible to others" (Shulman, 1986, p. 9).
Research-based Knowledge	
Situated Cognition - Knowing to Act	Situated Cognition - Knowing to Act 16 highlights the act of teaching is multi-dimensional in nature and that teachers' choices simultaneously reflect mathematical and pedagogical deliberation (Dapaepe et al., 2013, p. 22)
Student Outcomes	Demonstrates an understanding that teacher efforts are geared to improve.
Subject Specific	The respondent indicates that PCK is subject specific. PCK includes an understanding of how and why a subject is organized in a certain manner. Teacher understands what is important and what is peripheral (Shulman, 1986, p. 9).
Teacher Characteristics	
Teacher Goals	Teacher Goals Respondent demonstrates a desire to learn in her subject and the related pedagogy
Teacher Knowledge	Broad catch all category for undefined knowledge by respondent.
Training	Teacher training needed for PCK
Understands the Why	Deep understanding of content and its use
Vague	Unclear or unfocused response
Vygotsky	Describes a continuum or a zone of proximal development process.
Efficient	Looks to simplify subject
Real World Connections	Helps students learn how content is implemented
Student Centered	Focused on student learning

Appendix C: Question 1 Codes Assigned to PCK Components & Teacher - Expert Perspective Comparison

PCK Component and Codes (Bulleted)	Frequency	Teachers	Experts
1) Knowledge of Student Mistakes and Misconceptions • Previous knowledge • Student outcomes • Experience	33	13.2%	20.3%
2) Teacher Knowledge of Instructional Strategies & Representations • Accommodative • Representation • Student caring • Technology	62	24.8%	13.5%
3) Content and Subject Knowledge • Subject-specific • Understanding why the topic is important	53	21.2%	13.5%
4) Pedagogical and Teaching Knowledge • Efficient • Student centered	44	17.6%	10.8%
5) Curriculum Knowledge • Critical thinking • Real world connections	13	5.2%	14.9%
6) Assessment Knowledge • Student outcomes	17	6.8%	6.8%
7) Context Knowledge • Situated cognition • Knowing to act • Experience	10	4.0%	9.5%
8) Teacher Knowledge of Purpose and Orientation	18	7.2%	10.8%
N=	250	100%	100%

Appendix D: Question 2 Codes Assigned to PCK Components & Teacher - Expert Perspective Comparison

PCK Component and Codes (Bulleted)	Frequency	Teachers	Experts
1) Knowledge of Student Mistakes and Misconceptions	69	20.5%	20.3%
• Previous knowledge			
• Student outcomes			
• Experience			
2) Teacher Knowledge of Instructional Strategies & Representations	79	23.5%	13.5%
• Accommodative			
• Representation			
• Student caring			
• Technology			
3) Content and Subject Knowledge	101	30.1%	13.5%
• Subject-specific			
• Understanding why the topic is important			
4) Pedagogical and Teaching Knowledge	48	14.3%	10.8%
• Efficient			
• Student centered			
5) Curriculum Knowledge	3	0.9%	14.9%
• Critical thinking			
• Real world connections			
6) Assessment Knowledge	15	4.5%	6.8%
• Student outcomes			
7) Context Knowledge	2	0.6%	9.5%
• Situated cognition			
• Knowing to act			
• Experience			
8) Knowledge of Purpose and Orientation	19	5.7%	10.8%
N=	336	100%	100%